HANDOUT

CRITICAL THINKING

2022-2023 EDITION

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CRITICAL THINKING

INTRODUCTION

The term "Critical Thinking" denotes that pack of abilities that we need to use in order to distinguish a bad and a good argument.

Critical Thinking as a normative course

Critical thinking is about NORMATIVE CORRECTNESS (how to think), not about how we actually think, actual patterns of thinking: it's not a statistic view about human thoughts because how we think is a psychological fact.
We have logic that we can use as a theory of normative correctness of reasoning.
We can put standards on everything, (ex. Games theory on strategic ideas, choices theory...) and based on them we are rational if we follow those type of standards, if we make decisions along on these criteria.

HOMO LOGICUS = infallible from a logical point of view, capable of detecting every fallacy. A man like that doesn't exist because everyone makes mistakes sometimes.
HOMO ECONOMICUS = perfectly rational being with preferences that are complete, transitive...

THEORY OF HEURISTIC AND BIASES

The theory of heuristic and biases it's about how we reason and some biases we follow when we think. Practically, this is a realistic description of the process of human decision making.

Kahneman and Tversky, two psychologists, observed that when we face problems and reasoning we know there's a logical and mathematical solution but in most cases we follow heuristic solutions.
Why do we do this? Because the mathematical solution takes us more time to solve our problem. This shows how we always look for the fastest solution in our life.

There are different kinds of SYSTEMATIC BIASES:

Framing effect
A cognitive bias composed by two options in which to choose but none is completely good or completely bad. In this case it is very important to check how things are written, to be sure that the two options are not the same option written in different ways.

Example: in finance we are in front of two or more differences choices everyday: investing with the possibility of a failure but also the possibility of a great gain or not investing so we can keep our amount of money equal, without risks but also without gain.
Why do we fail in recognizing that the two programs are equivalent? Because problems are presented in a different way + role of emotions + system 1 overcoming system 2

Hindsight bias
It is about how a received information changes the perspective on an ex ante probability.
Indeed, it has been observed that the happening of an event in the present brings to a systematic overvaluation of the percentage that that event could have happened in the past.

Example: Who won the Nepales war? English with better weapons and military organization or Nepaleses with stronger motivation and better knowledge of the territory?

- ⇨ The British army won the war. Based on the hindsight bias and given the information about the victory, an individual will tend to overvalue the ex ante probability about an English win

Anchoring effect

The tendency for a person to rely heavily on the first piece of information they receive
Explanation: given figures as anchors, points of reference then adjustments.
If the figures are reliable indicators, anchoring works as a good heuristic; if not, it leads to error

ARGUMENTS

In everyday life people use arguments to persuade other people that a certain statement is true, reliable, probable or to prove the inverse thing.
What does distinguish argumentation from other linguistic practices?
<u>An argument is a connected series of statements (premises) that are intended to give reasons supporting a further statement (conclusion).</u>

We have some linkers that make us understand what an argument is.
The basic structure of an argument is:
Premises + conclusions + argumentative structure (connects the premises to the conclusion)

- ⇨ The argumentative structure makes it possible that the premises justify the conclusion

<u>Simple arguments:</u> two or more premises give reasons for a conclusion
<u>Complex arguments:</u> they are made up by two or more simple arguments connected to one another

  - First kind of complex argument: the conclusion of arg. A is a premise of argument B
  - Second kind of complex argument: arg A and arg B support the same conclusion

A good argument has true premises that give reasons for the conclusion being true, indeed it is a bad argument.

- ⇨ Check on the Internet the example of Brancusi case "*Bird in Space*" about this topic

Deductive arguments

Aims at providing conclusive reasons, to the effect that if the premises are true then the conclusion must be true.

- Valid deductive argument: A deductive argument is valid if and only if the truth of its premises guarantees the truth of its conclusion. Or equivalently if and only if the conclusion necessarily follows from the premises. It is a logic necessity: to affirm the premises and deny the conclusion would be <u>incoherent</u>.
- Invalid deductive argument= premises are true and yet the alleged conclusion false
- Sound argument = a deductive argument is valid and it has true premises

Non deductive arguments

In good non deductive arguments premises provide non conclusive reasons for the conclusion.
If the premises of a good non deductive argument are true they justify the belief that the conclusion is also true. A good non deductive argument may be better or worse, weaker o stronger depending on the degree with which it warrants the conclusion.

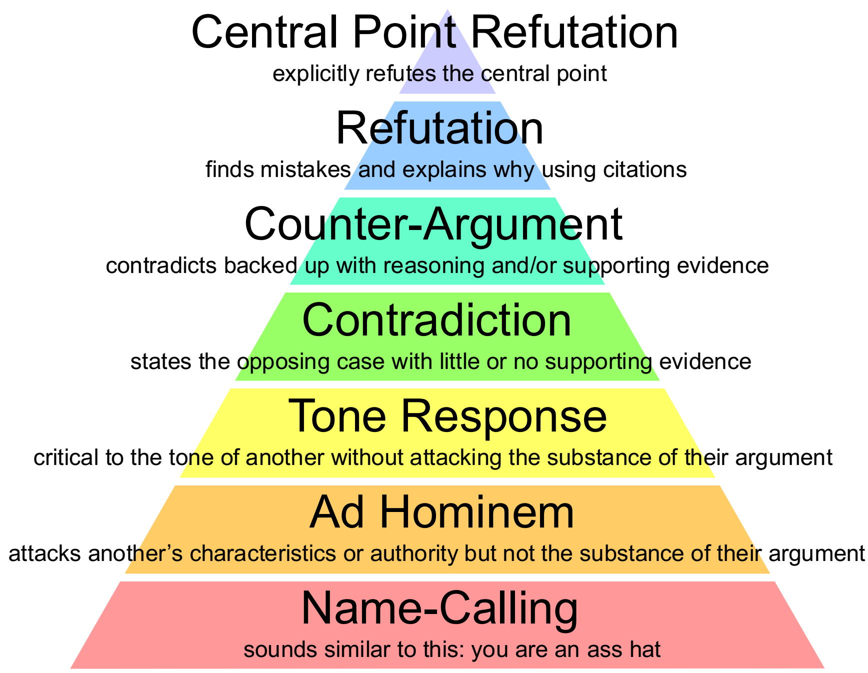
1. Induction: generalizing data
2. Abduction: making hypotheses
3. Analogy: establishing similarities

DISAGREEMENTS AND WAYS TO REPLY AN ARGUMENT

Rules of a rational discussion

1. The participants in a discussion must not prevent others from putting forward a thesis or raising objections
2. If someone attacks someone else's thesis then their attack must be directed against that thesis and not against the person
3. Whoever puts forward a thesis should defend it if others request this
4. You can defend your own thesis only by putting forward an argument in favor of it
5. The premise of an argument should not be presented as a position already accepted by all, nor should one deny a premise that has already been accepted during the discussion
6. A thesis can be considered successfully defended only if sound or sufficiently strong arguments have been proposed in its favor
7. If a thesis has been successfully defended, then others must withdraw their doubts regarding that thesis. If the defense of a thesis has not been successful, then whoever has advanced it must withdraw it

The pyramid of disagreement



LEVEL 0: argumentum ad baculum (of the stick) one threatens others to convince them: violation of rule 1,2
LEVEL 1: argumentum ad hominem=> attack to the person rather then the idea, violation of rule 2
LEVEL 2: reply ot the tone and not to the substance of the argument, violation of rule 2+ another fallacy: the straw man
LEVEL 3: simply denying the thesis advanced by the interlocutor without presenting an argument in support of this negation, false dichotomy. Violation of rule number 3
LEVEL 4: arguing for the opposite thesis. This is the first type of reaction in which the disagreement becomes significantly rational
LEVEL 5: confutation: it is shown that the other's argument is not good. There is at least a logically possible scenario the proposition is valid, therefore the conclusion of the initial argument does not follow from the premises

Let's analyze level 5 in more detail: to debunk an argument we can do that by…

1. Attacking the argumentative structure (logic)
2. Attacking the truth (or justification) of the premises

⇨ 1. To attack the argumentative structure you must find logical mistakes such as:
  - Unwarranted generalization: it may happen during the induction process
  - Circular reasoning: the conclusion is also one of the premises. That's the only case in which sound arguments are not good arguments. In other subtler cases of circular reasoning, even though the conclusion is not one of the premises, it is presupposed by one of the premises. That is: one of the premises is such that, in order to be true, the conclusion must be true

⇨ 2. Against the truth of the premises:
  - Counter examples
  - Reductio ad absurdum, reductio is a typical confutation procedure. A implies an absurdum, ergo non-A. Rigorously employed in mathematical reasoning, in which from a set of premises, the negation of one of them is deduced. In daily life it is used in a less rigorous way.

REASONING WITH CONDITIONALS

Deductive hypothetical sentence

If A and B and C and D and E are true, then I is true
What did we prove with our reasoning above?
That option I is true? NO.
We proved that: If assumptions A, B, C, D, E are true, then also I is true.
In other words, we proved the truth of the conditional sentence:
If A and B and C and D and E, then I
This is very different from proving I, of course.

Deductive hypothetical reasoning

In this kind of reasoning, we:

- Start by making one or more hypotheses / assumptions; (*in the example, assumptions A – E*);
- If opportune, we deploy other premises, under condition that we have proved or verified them before (*no such additional premise, in the above example);*
- Deduce a sentence following from the assumptions (*in our case: sentence I)*
- Conclude the conditional sentence (if… then) stating: If the assumptions are true, then what we have deduced is true (*in our example: If A and B and C and D and E, then I)*

This kind of reasoning is called deductive hypothetical reasoning, since:

- It is based on hypotheses, so assumptions that can be made just temporarily, for the sake of discussion, and with the purpose of showing that if the hypotheses would be/are true, then a given conclusion would be/is true.

Conditionals

For each pair of propositions A and B there is another complex proposition which is the conditional (or the implication) of A and B, in symbols A → B, where A is the antecedent of the conditional (or of the implication) and B the consequent of the conditional (or of the implication).
Example: The antecedent of a conditional is: "*Napoleon was German*"
The consequent of the conditional is: "*Napoleon was European*"
An example of a conditional statement will then be:
(⋆) "*If Napoleon was German then he was European*"

It is important to note that, if I say that, I am not committing myself to claim that antecedent and consequent are both true: we have seen this before; I am not forcing myself to support that Napoleon was German, nor that Napoleon was European.
Rather, what I am doing when I say "*If Napoleon was German then he was European*" is to maintain that in the hypothesis that the antecedent is true, "Napoleon was German" even the consequent will be, or it will be true that "Napoleon is European ".
Basically, I am committing to say that it is not the case that the antecedent is true and the consequent is false). That is:

If A → B is true, then it is not the case where A is true and B is false.
Hence, the truth of A → B excludes that: A is true and B is false.
That is: if it is the case that A is true and B is false, then A → B is false.

There are three possible combinations left, yet:
1. A is true and B is true
2. A is false and B is true
3. A is false and B is false

What's the truth value of A → B if one of situations 1 – 3 applies?
So-called material conditional provides a possible reply to the question.
Semantic truth-conditions:
The conditional A → B is false if the antecedent A is true and the consequent B is false, true otherwise.

A	B	A → B
T	T	T
F	T	T
T	F	F
F	F	T

In particular, the semantics of the material conditional states that:
A conditional statement A → B is:
1. false if and only if A is true and B is false;
2. true in all other cases.

The semantics of the material conditional is truth-functional, that is: the truth value of a sentence such as A → B is decided entirely by the combination of truth values of the antecedent (A) and of the consequent (B).

Notation: A ⇔ B is to be read: «A → B and B → A»
Oddities of the material conditional.
Why are we puzzled by this?
Because there is no connection between the antecedent and the consequent of the conditional. We tend to read "If... then" as if it presupposes a relevance of the antecedent for the consequent. Sometimes, we read it as something expressing a cause-effect connection.

Example: *If temperature rises, then ice melts.*
We tend to evaluate the truth of «if A then B» on the ground of these connections.
However, this depends on contents of A and B, not simply from their truth values.
This suggests that... Oddities of the material conditional REASONING
A (assumption)
...
...
B
$A \rightarrow B$
Example:
- *«Napoleon is German» (A) (assumption)*
- *«All Germans are European» (...)*
- *therefore: «Napoleon is European» (B)*
- *therefore: «if Napoleon is German, Napoleon is European» ($A \rightarrow B$)*

Conditional elimination (modus ponens)

Deductive rules for conditional reasoning

CONDITIONAL INTRODUCTION
A (assumption)
...
B

$A \Rightarrow B$

CONDITIONAL ELIMINATION
$A \Rightarrow B$
A

B

Fallacies of deductive reasoning involving conditionals.

- **The fallacy of asserting the consequent:**
  Example: *If Italy imposes medical tests on immigrants, then they want to ascertain their medical conditions. In Italy we want to ascertain the medical conditions of immigrants Therefore: Italy imposes medical tests on immigrants*
- **The fallacy of denying the antecedent**
  Example: *If you do standardized tests in the schools, then you wish to check the work done by the teachers. In Italy there are no tests in the schools. Therefore: In Italy we do not wish to check the work done by the teachers.*

Modus tollens

Example: *If you do standardized tests in the schools, then you wish to check the work done by the teachers. In Italy we do not wish to check the work done by the teachers. Therefore: In Italy we do not do standardized tests in the schools.*

Wason Task and Modus Tollens

Example: *Rule: "If there is the letter A on one side of a card, then there is the number 2 on the other side". Task: Indicate which cards you have to turn to determine if the rule is true or false.*

The task assigned to you is to identify which cards you need to turn to check whether this rule of construction is true or false.
The rule is a conditional one, as mentioned above:
If there is the letter A on one side of a card, then there is the number 2 on the other side.
The correct solution is to turn the card with A and the one with 5.

Modus Tollens
$p \rightarrow q$
$\neg q$
____
$\neg p$
The cards that can falsify the rule are:

- *The letter A (there is an A on one side but not the 2 on the other side)*
- *and the one with the number 5 (a number other than 2 has a letter A on the other side).*

The majority of the participants in the experiment give an incorrect answer, choosing in addition to the card marked with A, the one marked with 2.

CONTERFACTUALS

Counterfactual reasoning= <u>the capacity to imagine alternative scenarios (where something contrary to facts happens) and being able to reason about such a scenario</u>

Three abilities:

- The ability to imagine an alternative scenario which is different from the actual one
- The ability to imagine an alternative scenario differing from the actual one only in one respect and being, for the rest, as similar as possible to the actual one
- The ability to reason about this alternate scenario

Example
Number of counterfeit headphones around = profit lost by Sennheiser
Resulting counterfactual taken to be true:
All those who buy counterfeit headphones, would buy original Sennheiser headphones in absence of counterfeit headphones.
And yet the «possible scenario» in which all who buy counterfeit headphones buy original ones in absence of counterfeit ones is implausible

Why is counterfactual reasoning important?
Because of counterfactuals and other cognitive abilities:

- Learning from errors
- Taking decisions
- Having regrets
- Having remorses ecc…

Fault lines

When we mentally represent to ourselves a certain (actual) situation or course of events we tend to modify counterfactually some aspects and not others

- Exceptional events
- Actions
- Events under our control
- Obligations
- Time
- Cause

1. The majority of subjects tend to modify exceptional or anomalous events
2. None of the subjects tends to undo some non-exceptional event with a very low probability to occur
3. None of the subjects tends to imagine a counterfactual alternative where an improbable event occurs

Two kinds of counterfactuals conditionals

To reason counterfactually is to be able to determine the truth (or falsity) of counterfactual conditionals

1. Indicative conditionals
2. Counterfactual conditionals

1.RAMSEY TEST FOR INDICATIVE CONDITIONALS
If A, then B

- We evaluate a conditional with respect to our beliefs stock C: We use (or we should use) the following procedure:

(RT) Add A to C and judge whether B comes true

- If we already know that A is true, we do not have to modify C and we only judge whether B is true.

- If we do not know whether A is true, we hypothetically add A to C and judge whether B follows.

Ex.If North Korea will reinforce their nuclear program, the United States will intervene

2.RAMSEY TEST FOR CONTERFACTUAL CONDITIONALS

If A had been the case, then B would have been the case

- In the case of counterfactuals, we know that A is false
- Adding A to C would result in a contradiction

(RT+) Add A to C as an hypothesis, and then (i) do the minimal adjustments to maintain consistency (without modifying your hypothetical belief in the antecedent) and (ii) consider whether B comes out true in the new stock.

Ex. If North Korea had reinforced their nuclear program, then the United States would have intervened

Beliefs stocks and alternate scenarios

- A minimal adjustment to C is a way in which we represent a counterfactual scenario. It is intended to represent those counterfactual scenarios that are the most similar to the actual scenario.
- A counterfactual like: If A had been the case, then B would have been the case
    - is true, in case in all the most similar scenarios (to the actual one) in which A is true, B is true too
    - is false, in case in the most similar to the actual counterfactual scenario in which A is true, B is false

Counterfactual fallacies

- True consequents: for indicative conditionals the following principle holds TC "b". Therefore "if A then B", TC doesn't hold for counterfactuals conditionals
- Strengthening the antecedent: for indicative conditionals, the following principle holds: (SA) "if A then B" therefore "if A and X, then B"
- Contraposition For indicative conditionals, the following principle holds: (CON) "If A, then B". Therefore: "If not B, then not A"
- Transitivity: For indicative conditionals, the following principle holds: (TRANS) "If A, then B", "If B, then C". Therefore: "If A, then C"

Countefactuals and casuality

Imagine that you have to find out whether there is a causal relation between A and B, a good idea is to ask: "If A had not occurred, would B have occurred?"

(CT) A causes B if and only if if A had not occurred, then B would not have occurred

- In case "if not-A had been the case, then not-B would have been the case" is true, then we will say that B counterfactually depends on A
- Therefore, by (CT): A causes B if and only if B counterfactually depends on A
  The counterfactual alternatives we tend to imagine are not causally relevant

Example

Bill & Suzy: Bill and Suzy throw stones at a bottle. Suzy throws first, or maybe she throws harder. Her stone arrives first. The bottle shatters. When Bill's stone gets where the bottle used to be, there is nothing there but flying shards of glass.

"Suzy's throw causes the bottle's shattering" is true but "If Suzy had not thrown the stone, the bottle would not have shattered" is false

Bill & Suzy's story falsifies CT

HYPOTHESIS AND INFERENCES

Making hypotheses and selecting the best one

Problem ⇒ Some hypotheses

ex. *H1 CF is caused by "epidemic influences" that occur on entire districts from time to time*
H2 CF is contracted in overcrowded environments
H3 diet is a crucial factor in contracting CF
H4 The difference is due to the fact that inexperienced medicine students train in FOC. The midwives working in the SOC follow better procedures and don't get the same risk of hurting patients
E1 INITIAL EVIDENCE there is a considerable difference in the infection rate in the FOC and in the SOC
E2 there's the same amount of people I the SOC and FOC
E3 foc and soc apply the same
This proves H1-H3 are wrong
E4 medicine students and midwives use exactly the same methods in examining patients in the two clinics
E4 proves H4 is wrong

Semmelweiss hypothesis

H5 medicine students working in the FOC carry infectious materials. They go from the autopsy room to the FOC without any sanitization.
Confirming evidence
E5 a doctor from the hospital wounded himself with a scalpel he was using in the autopsy room, he developed the same symptoms that were observed in patients having childbed fever
E6 after the students had to wash their hands with a bactericide solution, the infection rate in the FOC went down, gradually converging with the infection rate of the soc

Inferences: deductive, inductive, abductive

Deduction preserves the truth of the premises, but an explanation is seldom purely deductive.
To be explained, surprising phenomena need some "ampliative inference"

Non-deductive inferences (ampliative) draw conclusions that are not certain – it can happen that the premises are true, the inference is good, and yet the conclusion is false.
Abduction and **induction** are non-deductively valid inferences
Three features they share:

1. <u>Ampliative</u>. The information provided by the conclusion is not contained or implicit in the information contained in the premises.
2. <u>Defeasible</u>. The evidence provided by true premises may well be very robust, and yet the conclusion may be false.
3. <u>Non-monotonic</u>. Addition of new evidence can lead us to drop conclusions that we had previously drawn

An inference is **inductive** if (and only if)

- it is not deductively valid
- its premises provide evidence in support of the truth of the conclusion.

Example:(P1) *All ravens observed thus far are black.(C) The next raven that we will observe is black.*
An inference is **abductive** if (and only if)

- it is not deductively valid
- its conclusions provide a plausible explanation for the premises.

Example:(P1*) A fellow student of mine does not find his smartphone after leaving the classroom.(P2) He had got his smartphone when in the classroom.(C) He left his smartphone in the room*

Inference to the best explanation

The kind of abductive reasoning exemplified above helps us make hypotheses on the ground of evidence, but it cannot contribute to control the hypotheses. In consequence, it does not contribute to rank different rival hypotheses. However, the hypothesis control step is crucial.
In particular, we wish that the selected hypothesis is the best explanation of the evidence available.
In order to do this, we deploy the so-called inference to the best explanation (IBE).

Four steps of IBE:

1. Inferring from the evidence a plausible explanatory hypothesis (reasoning from evidence)
2. Testing the hypothesis with new evidence (reasoning from hypothesis to evidence)
3. Comparing the hypothesis with rival explanations
4. Assessing the hypothesis with the relevant criteria

- Quantity Test: if H1 explains n items of evidence and H2 explains m where m > n, then H2 > H1
- Quality Test: if H1 explains less important items of evidence and H2 explains more important ones, thenH2 > H1
- Prediction Test: if H1 does not successfully predict new evidence and H2 does so, then H2 > H1More analytically:(3.1) No prediction < successful prediction(3.2) Unsuccessful prediction < successful prediction(3.3) No prediction > unsuccessful prediction?
- Hard to state second-order tests

Reasoning from effects to causes

Example
Student Health Service of Cornell University, mid-March 1968.

- *5 medical students display flu-like symptoms (headaches, fever, muscle aches); a blood test reveal they have toxoplasmosis.*
- *The 5 students share no relevant space, and hardly know one another.*
- *All 5 students ate rare hamburgers at the dormitory snack bar on a given night, and nothing else.*
- *The doctors conclude that: The ingestion of rare meat CAUSED transmission of toxoplasmosis in the five cases.*

The doctors are
1. engaging in causal reasoning
2. reasoning from effects to causes:
   - the conclusion of the reasoning is a <u>causal statement (X caused Y)</u>
   - . their premises are just factual statements. Relevance in everyday life: trials, economic explanations, public debate, etc

5 methods of causal reasoning

Introduced by John Stuart Mill, conclusive in eliminating hypotheses on causal connections.
They can confirm (albeit provisionally) hypotheses on causal connection
All the methods start with
(i) An effect Y whose cause is to be identified;
(ii) A number of relevant prior factors A, B, C, D, ... that could be the cause of Y.
Three crucial features of the methods:

1. They do not help discover new facts;
2. They do not offer conclusive proofs about causal connections;
3. They control hypotheses on what causes what

1. Method of agreement (necessary conditions)

Four friends go out together for dinner, but when they go back home they all start feeling sick and experience stomach aches
<u>X is a necessary condition for Y if and only if Y does not occur in absence of X.</u>

The method goes as follows: If in all cases E1, ..., En where effect Y occurs, there is a single prior factor X that is shared by all the cases, then X is the cause of Y

2. **Method of difference (sufficient conditions)**

X is a sufficient condition for Y if and only if X suffices to determine Y.
The method goes as follows: Given cases E1, ..., En, if effect Y occurs in all cases but some Ei, and all the cases but Ei share all factors, then the factor that just Ei does not share is the cause of Y

3. **The joint method (of agreement and difference)**

It applies the two previous methods at once:
Given cases E_1, ..., E_n, if effect Y occurs in all cases but some E_i, and prior factor X is the only factor they share, then X is the cause of Y.
The methods picks necessary and sufficient conditions of Y.

Features and limits of Mill's methods

Selection of prior relevant factors is not always trivial.
Problem: causation may well be indeterministic. Mill's methods are ok when it comes to deterministic causation, but they can give wrong results when they deal with indeterministic causes.

STATISTICAL REASONING

A statistical generalization stems from information concerning a subset of (randomly chosen) individuals to arrive at a (probable) conclusion regarding the composition of the whole set.
Sample à Population (inductive reasoning)

- ⇨ This is the kind of reasoning commonly used to draw general conclusions based on opinion surveys

1) Sometimes the argument is purely statistical. In this case, we start from the fact that a certain percentage of our sample has the property P and we reach the conclusion that the same percentage of the whole population has the property P
2) Sometimes the argument does not concern just present objects but also future objects. In this case, we draw a conclusion on future cases based on past cases. This kind of arguments presuppose that the world is uniform and governed by laws.

Basic terminology

Suppose that we want to know who the most popular candidate in the next general election is.

- Suppose that the researchers ask 1000 people whether they like candidate x (**SAMPLE**).
- 371 people belonging to the sample reported that they like candidate x.
- The researchers concluded that about 30% of the entire adult population likes candidate x (**STATISTICAL PROJETION**).
- Since the other candidates have received less appreciation, it is drawn the conclusion that candidate x is the most popular (**GENERALIZATION**)
- **Target property**: the object of investigation (the most popular candidate)
- **Measured property**: what we look for in order to know about the target property (how many people report to appreciate a certain candidate)
- **Margin of error**: the maximum amount by which the sample results are expected to differ from those of the actual population (e.g.: 3%)
- **Accuracy**: how good is the measured property as an indicator of the target property

The basic principle

● Statistical generalizations are based on the following demonstrable mathematical theorem: if the percentage of the members of a set S that have the property P is x, then in most sufficiently large subsets of I a similar percentage of members has this property.
● This is a mathematical fact. However, statistical generalizations can be weakened by a number of factors.

Relationship between the measured property and the target property, called ACCURACY, must be meaningful and sample must be adequate

The sample

● The sample must have a sufficient size. Otherwise, the generalization is unwarranted.
● Generally, the larger the sample, the smaller the margin of error of our result.
● However, the relationship between the size of the sample needed to have good quality generalizations and the size of the population is not constant
● When the population is very large (>100,000), the cardinality of the sample can remain constant as the population increases, even if one wishes the same margin of error

The representativeness with respect to the entire population also affects the correctness of an argument containing statistic s
● A non-representative sample can be biased if it Is chosen in order to arrive at a certain conclusion
.●Individuals in the sample should be chosen randomly from the population considered

Representativeness of the sample

Sometimes mistakes about the representativeness of the sample are very subtle.
Example
In 1936 a magazine named Literary Digest sent 10 million questionnaires asking which candidate for the presidential election the recipient would vote for: Franklin Roosevelt or Alf Landon. It received 2.5 million returns. Given the largeness of the sample, the magazine confidently predicted that Landon would win. The sample was randomly drawn from names in telephone books. Therefore, it seemed to be representative. Actually, in 1936 there were just 11 million payphones in the United States, and many of the poor – especially the rural poor – did not have payphones. A large percentage of these underrepresented groups voted for Roosevelt, the Democratic candidate. Even though the sample seemed to be representative since it was randomly chosen, actually it was not.

Evaluating arguments containing statistics/question

Even if the sample is chosen appropriately, there may be other problems, such as a bias in asking the question.
Example:
A survey promoted by the National Rifle Association (NRA)concerning the opinions of Americans with regard to gun control in the USA. What do you prefer?(a) to preserve the constitutional right to keep and bear firearms, or(b) to leave helpless citizens at the mercy of criminals?
In these case too, mistakes can be subtler. If I am answering to a survey on the products of company X, knowing that it is company X that conducts the survey, I might be led to give a more positive review of X's products than that I would have given if I had not known who the interviewer is.

Statistical generalizations and daily life

Statistical generalizations are very common in daily life, we often make the same mistakes we have seen above:
○ We generalize from very small samples
○ Samples are often badly chosen

Strong and weak inferences

The more accurate the choice of the sample is, the larger the sample is, the more significant the connection between measured property and target property, etc. the stronger our inference becomes.
● The larger is the declared error margin, the stronger is our inference (even though is less informative)
=> *For instance, concluding based on the sample that candidate x is preferred by 37± 3% of the voters is stronger but less informative than concluding that x is preferred by 37± 10% of voters.*
●If 100% of the members of our sample have the property P, then we can draw universal conclusions about the population under examination

Generalizations about the future

When the relevant population concerns future cases, then necessarily our sample cannot be randomly chosen and then it might not be representative. These inferences presuppose that past and future are uniform. However, this uniformity cannot be justified on mathematical premises alone.
Scientific laws are based on such generalizations

Simple induction

or simple predictive inference is a generalization of this kind:
Every A observed so far has the property => The next A will have the property P

Economic laws

Many laws of economics are generalizations of this kind.

- They are grounded on the behavior of economic systems in the past and predict the behavior of economic systems in the future.
- As other predictions of this kind, these predictions presuppose the uniformity of the behavior of economic systems over time .
- In the past, a rise in the interest rate has produced a reduction in inflation, but also a decline in investment Is If we increase interest rate, the inflation will fall but also investments will decline.

Statistic applications or statistical syllogism

Sometimes we make inferences that apply statistical generalizations to particular cases.
Such applications go in the reverse direction with respect to statistical generalizations:

○ Statistical generalizations: from a sample to the entire population
○ Statistical applications: from the entire population to a sample.

95% of graduates fluently read complex texts Ann is a graduated Ann fluently reads complex texts

Notice that:

● The higher the percentage in the first premise is, the stronger the argument is.
● The first premise is (usually) a statistical generalization. Since it is a conclusion of an induction, its truth is not warranted.

REFERENCE CLASSES: A potential problem for statistical syllogisms can stem from the reference class to which the object under examination is ascribed. Statistical applications can result in different conclusions. One way of dealing with competing statistical applications is to combine the reference classes

PROBABILITY RULES

Probability

Roughly speaking, probability is a numerical description of how likely an event is to occur or how plausible it is that a proposition is true.

Examples of probabilistic statements are the following:
"*69.1% of all individuals under the age of 60 who contracted the SARS-CoV-2 infection did not develop clinical symptoms*". Medicalxpress
"*Pimco, the bond fund giant, says there is a 40 per cent probability that Britain will vote to leave the EU in the upcoming referendum, an outcome that could have a "long-lasting" market impact*". FT

The notion of probability has different interpretations:

Classical definition
The probability (P) of an event E is the ratio between the number of favorable cases and the number of possible cases, provided that the latter are equally possible.
So, if the possible cases are n and the favorable cases are nE, according to the classic definition, the probability of the event E occurring will be: P(E) = nE /n.
The guiding idea is that probability is shared equally among all the possible outcomes, so that the classical probability of an event is simply the fraction given by the number of favorable cases and the total number of possibilities in which the event occurs.
Example: *The probability of getting the number 6 in drawing a fair dice is 1/6.*

Frequentist definition of probability
The probability (P) of an event E is the limit of its (relative) frequency of successes, i.e., of occurrence of the event, when the number of tests tends to infinity: P(E) = $\lim_{n \to \infty} nE/n$
Advantage: it can be applied to cases in which the space of possibilities is not equally distributed among the totality of events
Problem: It is difficult to assign a probability to unique events!

Example: *The probability of the onset of lung cancer in a population on non-smokers among young adults in Italy is 0.02.*

Frequentism

The event space Ω is the set of all possible results of a probabilistic trial (e.g., tossing a coin).
An event is a whatever set of the results of a probabilistic trial.
The probability of an event, in a frequentist tradition, is the relative frequency of the set of results of a statistical trial associated with the event, evaluated on an (ideal) infinite number of repetitions of the trial.

Subjectivism

According to the subjective (a.k.a. subjectivist) conception of probability, probability measures the degrees of confidence or partial beliefs of suitable agents for the occurrence or not of an event. The probabilities are therefore subjective by definition and reveal the attitude to accept or refute specific bets.
Example: *Spain has the probability 0.2 to win next football world cup.*
Subjective does not mean arbitrary: the degree of confidence can be grounded on rational beliefs.

Advantage: it can also be applied to unique events.
Problem: it is often difficult to assign a numerical value to our degree of confidence.

Probability calculus rules

- $\{A \cap B\}$ means that A and B occur together
- {¬A} means the complementary event of A
- {A ∪ B} means that A or B occurs.
- {B|A} means B will occur based on the assumption that A has already occurred
- The probability P of an event E, indicated as P(E), satisfies the following condition: $0 \leq P(E) \leq 1$
- P(Ω)= 1. Ω is the probabilistic space of all possible alternatives.

Intersection

- If two events A and B are independent, then P(A∩ B) = P(A) x P(B)
- In general, if $A_1,\dots,A_k$ are independent events, then: $P(A_1 \cap A_2 \cap A_3 \dots, A_k) = (P(A_1) \times P(A_2) \times P(A_3) \times .. P(A_k))$
- If two events A and B are not independent (i.e., are dependent) the formula becomes: P(A ∩ B) = P(A) x P (B|A) hence: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Examples

- The probability of tossing two heads on a single throw of two coins is the following: If H1 and H2 are two occurrences of head, then P(H1 ∩H2) = 1/2 x 1/2 = 1/4

- A and B designate the event of drawing two kings from a deck when the first card is not replaced before the second is drawn.

- *P(A) = 4/52*
- *P(B|A)= 3/51*
- *P(A ∩B) = (4/52) x (3/51)=12/2652 = 1/221= 0.004...*

Notice that if A and B are independent P(B|A) = P(B); therefore, this version of the formula is more general.

Union

- If the results of a statistical trial, indicated with A and B, are two events that cannot occur together (mutually exclusive), then P(A ∪ B) = P(A) + P(B)
- Whether or not the event A and the event B are mutually exclusive, the general rule for disjunction is: P(A ∪ B) = P(A) + P(B) – P (A ∩ B)
- Assuming also that A and B are independent, we have: P(A ∪ B) = P(A) + P(B) – (P(A) x P (B))

Examples

- the probability of drawing either a king (K) or a queen (Q) (of any suit) from a deck of cards on a single draw is: P(K ∪Q) = 4/52 + 4/52= 8/52= 2/13= 0.15

- Consider the probability of getting at least one number 6 (indicated by «S») when rolling a pair of dices: P(S1 or S2): 1/6 + 1/6 – (1/6 x 1/6)= 2/6 – 1/36 = 0.30

Notice that if A and B are exclusive, the formula simplifies because P (A ∩ B) = 0

If we sum all the elements in A and all the elements in B, we sum the elements in the intersection twice. Hence, we must subtract P (A ∩ B) to P(A) + P(B).

Negation

- Negation rule: P(A ∪ ¬A)= 1, but since A and ¬A are exclusive, then P(A) + P(¬A) = 1, hence:
  P(¬A)= 1 - P(A)

As we have seen, the conditional probability is defined as: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

- If the events are independent, then: P (B|A)= P (B) = P (B|¬A)
- If two events are dependent, then: P (B|A)≠ P (B) ≠ P (B|¬A) and P(A ∩ B) ≠ (P (A) x P (B))

Bayes Theorem

Bayes theorem is a fundamental theorem of probability calculus. It allows us to update our probabilistic values once new information is gained.
Let us consider two events mutually exclusive and jointly exclusive. Jointly exclusive means that at least one event should necessarily occur.

$P(A \cap B) = P(A) \times P(B|A)$

$P(B \cap A) = P(B) \times P(A|B)$

$P(A) \times P(B|A) = P(B) \times P(A|B)$

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(B)}$$

P(A) = initial (prior) prob.; P(B|A) = likehood; P(A|B)= posterior prob.
But $B = (A \cap B) \cup (\neg A \cap B)$

$$\text{Then } P(A|B) = \frac{P(A) \times P(B|A)}{P(A) \times P(B|A) + P(\neg A) \times P(B|\neg A)}$$

Probabilistic biases

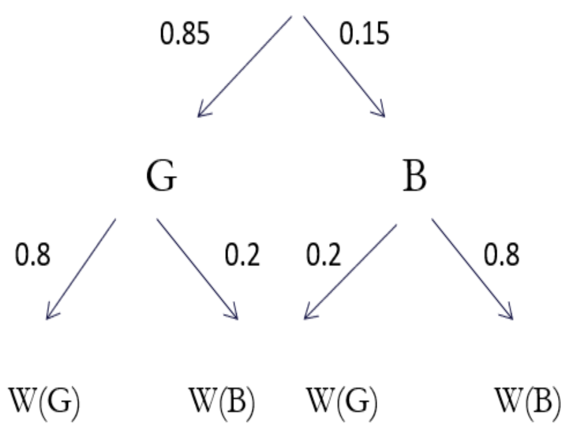
EXAM AND DISEASE Suppose a disease affects 2% of people. Suppose a person undergoes a medical examination aimed at diagnosing the disease. Assume that the test has an 80% reliability. How much does the positive test increase the probability of being affected by the disease?
Obviously, that positive test increases the probability that that person has the disease but perhaps less than one might expect. P(D) is the probability that the subject has the disease prior to the test (0.02). Thus, P(¬D) is 0,98
P(D |T) is the probability that the subject has the disease given a positive outcome of the test.
P(T |D) denotes the likelihood that the test gives a positive outcome provided that the subject has the disease (0.8 because the test is 80% reliable).
P(T|¬M) indicates the likelihood of being positive to the test despite not having the disease (0.2).

$$P(D|T) = \frac{P(D) \times P(T|M)}{P(D) \times P(T|D) + P(\neg D) \times P(T|\neg D)}$$

$$= \frac{0.02 \times 0.8}{(0.02 \times 0.8) + (0.98 \times 0.2)} = 0.075 = 7.5\%$$

Cab problem

- A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. 85% of the cabs in the city are Green and 15% are Blue.
- A witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.
- What is the probability that the cab involved in the accident was Blue rather than Green knowing that this witness identified it as Blue? B=the cab is blue ; G= the cab is green; WB= the cab is blue according to the witness; WG= the taxi is green according to the witness



Prosecutor fallacy

Suppose a city has a population of 5,000,000 people. Suppose one of them commits a crime. A DNA sample of Jones is taken, and it is verified that it matches a biological trace found at the crime scene. The DNA test only fails in one in 10,000 cases. Reasoning of the prosecutor: there is only a 1/10,000 chance that Jones is innocent. Since this is a very small , we have sufficient evidence that Jones is the culprit. Using the language of conditional probability, the prosecutor is saying the following: Let T = DNA test is positive. let C = the defendant is culprit. Now the prosecutor calculates P(T|C), i.e., the probability of a positive test, given that Jones is culprit. However, the jury must consider the conditional probability P(C|T) . That is, what is the probability of guilt given that the defendant's test is positive? Before the test, P(C) is 0.0000002 (there are 5 milion people living in the city); P(T|C)=0.9999 while P(T|¬C) is 0,0001. Applying Bayes' theorem, we can calculate that P(C|T) = 19,96%. A too small percentage to sentence Jones!

Risk and uncertainity

Risk is the probability "that a particular adverse event occurs during a stated period of time, or results from a particular challenge" (Royal Society 1992, 2).
The Royal Society takes risk here "as a probability in the sense of statistical theory", which obeys "all the formal laws of combining probabilities".

Fundamental (severe or deep) uncertainty is not probabilistic and exemplifies the mostly common form of uncertainty that we experience in daily life.
We do not know the probability function for this type of uncertainty
Fundamental (severe, genuine, great) uncertainty exemplifies common forms of uncertainty for which is difficult to formulate meaningful probabilistic evaluations.
By 'uncertain' knowledge I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty [...]. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence [...]. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know (Keynes 1973: 113–114).

Unknown unknows

Known knowns: conditions of certainty (we perfectly know the occurrence of events and their consequences)
Known Unknowns: conditions of risk (we call "list ex ante" all events and we can evaluate them with meaningful probabilistic measures)
Unknown Unknowns: fundamental uncertainty (events are listable ex ante but we are not able to evaluate them from a probabilistic perspective).

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