BIEM

A.Y. 2024/2025

B.LAB

HANDOUTS

MATHEMATICS MODULE II (APPLIED)

-FORMULAS-

WRITTEN BY
MICHELE ROSSINI

TEACHING DIVISION

This handout is written by students with no intention of replacing university materials.

It is a useful tool for studying the subject, but does not guarantee preparation as exhaustive and complete as the material recommended by the University.

Mathematics Module II (Applied)

Formulas

Michele Rossini - BIEM15 - AY: 2023-2024

This handout has been written by a student with no intention to substitute the University official materials. Its purpose is to be an instrument useful to the exam preparation, but it does not give a total knowledge about the program of the course it is related to, as the materials of the university website or professor.

Probability

Uniform Probability: Ω finito $P(\{\omega\}) = \dfrac{1}{|\Omega|}$

Dirac Probability: Ω finito $P(A) = \begin{cases} 1 & if \quad \omega_0 \in A \\ 0 & if \quad \omega_0 \notin A \end{cases}$

Poisson Probability: $P_n = e^{-\lambda} \cdot \dfrac{\lambda^n}{n!}$

Geometric Probability: $P_n = (1-q) \cdot q^n$

Expected Value: $E_p(f) = \sum_{\omega \in SuppP} P(\omega) \cdot f(\omega)$

Variance:

$$V_p(f) = E_p(f(\omega) - E_p(f))^2$$

$$V_p(f) = E_p(f - E_p(f))^2$$

$$V_p(f) = E_p(f^2) - E_p(f)^2$$

$$V_p(\alpha f + \beta) = \alpha^2 V_p(f)$$

Standard Deviation:

$$\sigma_p(f) = \sqrt{V_p(f)}$$

$$\sigma_p(\alpha f + \beta) = |\alpha| \sigma_p(f)$$

Covariance:

$$COV_p(f, g) = E_p(f(\omega) - E_p(f))(g(\omega) - E_p(g))$$

$$COV_P(f, g) = E_p(f \cdot g) - E_p(f) \cdot E_p(g)$$

$$COV_p(f, f) = V_p(f)$$

$$COV_p(f, k) = 0 \quad \forall k \in R$$

$$COV_P(\alpha f + \beta, \gamma g + \delta) = \alpha\gamma COV_P(f, g)$$

$$COV_p(f + g, h) = COV(f, h) + COV_p(g, h)$$

Pearson Coefficient $$-1 \leq \frac{COV_P(f,g)}{\sigma_p(f)\cdot\sigma_p(g)} \leq 1$$

Distribution Function: $$\Phi(x) = P(f \leq x)$$

Simple Density Function: $$\Phi(x) = \sum_{i:y_i \leq x} \varphi(y_i)$$

Integrable Density Function: $$\Phi(x) = \int_{-\infty}^{x} \varphi(t)dt$$

Expected Value: $$E_p(f) = \int_{-\infty}^{+\infty} x d\Phi(x)$$

Cavalieri formula: $$E_p(f) = \int_{0}^{+\infty} (1-\Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx$$

If $\Phi(x)$ is a **distribution function** you have to verify:

- $\Phi(x)$ positive
- $\Phi(x)$ increasing
- $\Phi(x)$ takes values in $[0,1]$
- $\lim_{x\to-\infty} \Phi(x) = 0$ and $\lim_{x\to+\infty} \Phi(x) = 1$
- $\Phi(x)$ continuous so

  $\lim_{x\to a^-} \Phi(x) = \lim_{x\to a^+} \Phi(x) = \Phi(a) = 0$ and $\lim_{x\to b^-} \Phi(x) = \lim_{x\to b^+} \Phi(x) = \Phi(b) = 1$

If $\varphi(x)$ is a **density function** you have to verify:

- $\Phi(x)$ positive
- $\Phi(x)$ continuous

  $\lim_{x\to a^-} \varphi(x) = \lim_{x\to a^+} \varphi(x) = \varphi(a) = 0$ and $\lim_{x\to b^-} \varphi(x) = \lim_{x\to b^+} \varphi(x) = \varphi(b) = 1$
- $\int_{-\infty}^{+\infty} \varphi(t)dt = 1$

Calcola expected value, Variance, covariance, Pearson coefficient, Gauss distribution e density, Uniform density, Poisson, Dirac

Financial Calculus

	SIMPLE Law	COMPOUND Law
PV (Discount)	$A = \frac{S}{(1+it)}$	$A = \frac{S}{(1+i)^t}$
FV (Accumulation)	$M = C(1+it)$	$M = C(1+i)^t$
***i* (Interest)**	$i = f(1) - 1$	$i = e^{\delta} - 1$
Force of Interest	$\rho(t) = \frac{f'(t)}{f(t)} = \frac{1}{1+it}$	$\rho(t) = \frac{f'(t)}{f(t)} = ln(1+i)$

Compound Law: $f(t) = (1+i)^t \Longrightarrow f(t) = e^{\delta t}$

EQUIVALENT RATES

Simple Interest: $i_m = \frac{i}{m}$

Compound Interest: $i = (1+i_m)^m - 1$

Nominal Annual Rate: $j_m \equiv m \cdot i_m$

COMPOUND CAPITALIZATION

Discounted Cash Flow: $DCF(x) = G(x) = \sum_{s=0}^{n} \frac{a_s}{(1+x)^{t_s}}$

Net Present Value: $NPV(i) = \sum_{s=0}^{n} \frac{a_s}{(1+i)^{t_s}}$

Investment: To find the IRR (internal rate of return), you need to $NPV(i) = 0$, that i is the IRR.

Loan: To find the EC (effective cost), you need to $NPV(i) = 0$, that i is the EC

ANNUITY

	Ordinary Annuity	Due Annuity
PV	$V_0 = R \cdot a_{n\neg i}$ $a_{n\neg i} = \frac{1-(1+i)^{-n}}{i}$	$V_0 = R \cdot \ddot{a}_{n\neg i}$ $\ddot{a}_{n\neg i} = a_{n\neg i}(1+i)$
FV	$V_T = R \cdot s_{n\neg i}$ $s_{n\neg i} = \frac{(1+i)^n - 1}{i}$	$V_T = R \cdot \ddot{s}_{n\neg i}$ $\ddot{s}_{n\neg i} = s_{n\neg i}(1+i)$

PERPETUITY

	Ordinary Perpetuity	Due Perpetuity
PV	$V_0 = \frac{R}{i}$	$\ddot{V}_0 = R\frac{1+i}{i}$

FIXED INCOME BONDS

ZERO COUPON BONDS:

Yield to Maturity (YTM): $i(0,T) = (\frac{N}{P_0})^{1/T} - 1$

$$i(t,T) = (\frac{N}{P_0})^{1/T-t} - 1$$

Duration: $D = Maturity$

COUPON BONDS:

Yield to Maturity (YTM): $G(x) = 0$

$$G(i) = -P_0 + \frac{a_1}{(1+i)^{t_1}} + \frac{a_2}{(1+i)^{t_2}} + \ldots + \frac{a_n + R}{(1+i)^T} = 0$$

Duration: $D = \sum_{s=1}^{n} t_s \cdot \frac{a_s}{P} \cdot \frac{1}{(1+i)^{t_s}}$

Price of the bond: $P(0,i) = \sum_{s=1}^{n} \frac{a_s}{(1+i)^{t_s}}$

Volatility: $$\frac{\Delta P(i)}{P(i)} \approx \frac{P'(i)}{P(i)}\Delta i = -\frac{D(i)}{1+i}\Delta i$$

FINANCIAL IMMUNIZATION

$V(z,i) = R(z,i) + P(z,i)$

$R(z,i)$ = **reinvestment** (up to z) is that part of the value (at z) that is given by the reinvestments of all the amounts corresponding to maturities preceding z

$P(z,i)$ = **discounted value** of all the future amounts paired at maturities from z on.

If $i\uparrow$ then $R\uparrow$ and $P\downarrow$

PRICING THEORY

Assets: Ex: $y_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $y_2 = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ $y_3 = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$ $y_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

States of the world: Ex: w_1= recession, w_2= stasis, w_3= growth

The number of elements of an asset corresponds to the number of state.

Ex: If state w_1 is reached, the asset y_1 will repay -1.

Portfolio: Ex: $\underline{x} = [1, -1, 1, 2]$

Asset Price vector: Ex: $\underline{p} = [1,2,1,3]$

Payoff of the Portfolio
Or **contingent claim**: $\underline{w} = \underline{x} \cdot \underline{y}$

Replicable claim: $\underline{w} = \sum_{j=1}^{n} \underline{x}_j \underline{y}_j$

Market: $W = spanL$

Complete Market: $W \equiv R^k$

Pure contingent claims: $\{\underline{e}_1, \ldots, \underline{e}_n\}$

Value of the Portfolio: $v(\underline{x}) = \underline{p} \bullet \underline{x}$

LOP: $R(\underline{x}^*) = R(\underline{x}^\bullet) \Longrightarrow v(\underline{x}^*) = v(\underline{x}^\bullet)$

Price of the claim: $p_{\underline{w}} = v(\underline{x})$

1st pricing formula: $p_{\underline{w}} = \sum_{j=1}^{n} p_j x_j$

2nd Pricing Formula: $p_{\underline{w}} = \sum_{j=1}^{n} \alpha_j P_{\underline{w}_j}$ contingent claims in terms of other contingent claims

Pricing Kernel: $f(\underline{w}) = \underline{\pi} \bullet \underline{w}$

Price of the pur contingent claim that corresponds to the state s_i $p_{\underline{e}_i} = f(\underline{e}_i) = \underline{\pi} \bullet \underline{e}_i = \pi_i$

Arbitrage of I kind: $\begin{cases} Y \cdot \underline{x} = R(\underline{x}) \geq 0 \\ \underline{p} \bullet \underline{x} = v(\underline{x}) < 0 \end{cases}$

Arbitrage of II kind: $\begin{cases} Y \cdot \underline{x} = R(\underline{x}) > 0 \\ \underline{p} \bullet \underline{x} = v(\underline{x}) \leq 0 \end{cases}$

FOR DOUBTS OR SUGGESTIONS ON THE HANDOUTS

MICHELE (MIKE) ROSSINI

michele.rossini@studbocconi.it
@mikerossinii
+39 3318814946

FOR INFO ON THE TEACHING DIVISION

VITTORIA NASONTE

vittoria.nasonte@studbocconi.it
@_vittorian_
+39 3274441476

ELENA CACIOLI

elena.cacioli@studbocconi.it
@elenacaciolii_
+39 3928931605

TEACHING DIVISION

OUR PARTNERS