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B.LAB

HANDOUTS

MACROECONOMICS

-SECOND PARTIAL-

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Chapter 15 - Financial mkts and expectations

☑ Reviewed ☐

📈 Macroeconomics: Chapter 15 – Financial Markets and Expectations

Reference: Lectures 16, 17, 18 (20–25 March 2025) and Chapter 15 Textbook (pp. 321–337)

Context: Extensions – Expectations. We now explore how expectations determine asset prices (bonds, stocks), moving beyond the IS-LM model's static treatment of interest rates.

> 💡 **Key message:** Expectations determine bond and stock prices. Long-term rates reflect the average of current and expected future short-term rates. Stock prices equal the present value of expected future dividends.

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1. Bond Prices and Bond Yields

What is a bond?

A bond is a promise to make one or more payments in the future. Bonds differ along two key dimensions:

- **Maturity** → the length of time over which the bond promises to make payments to the bondholder.
- **Risk** → two types:
  - Default risk: the issuer (government or firm) will not pay back the promised amount. Government bonds carry lower default risk than corporate bonds.
  - Price risk: uncertainty about the price at which you can sell the bond before it matures.

Considering maturity, the key definitions are:

- **Yield to maturity** (or simply *yield*): the **interest rate associated with bonds of different maturities.**
- **Short-term interest rates**: yields on bonds with maturity of one year or less.
- **Long-term interest rates**: yields on bonds with maturity longer than one year.

Bonds can also be classified by payment structure:

- **Discount bonds** (zero-coupon bonds): **one single payment at maturity**, called the **face value.**
- **Coupon bonds**: **multiple payments before maturity** (*coupon payments*) **plus a final payment at maturity** (face value).
  - the ***coupon rate*** is the ratio of coupon payments to face value
  - the ***current yield*** is the ratio of coupon payments to the bond price.

    → Neither is the right measure — what matters is the **yield to maturity**.

Bond quality is assessed by **rating agencies** (Standard & Poor's, Moody's).

The **risk premium** is the difference between the rate on a given bond and the rate on the highest-rated bond. Bonds with high default risk are called **junk bonds**.

The Price of a Discount Bond

We focus on maturity, setting risk aside for now. For a zero-coupon bond, the price $\$P_B$ is simply the present discounted value of the future payment:

$$\$P_B = \frac{\text{Payment in the future}}{1+i}$$

The lower i, the higher the bond price. This inverse relationship is the foundation of bond pricing.

> **Example:** A bond promises $100 in one year and costs $95 today. Then $i = \frac{100-95}{95} = 0.05 = 5\%$.

For bonds of different maturities (all promising $100 at maturity), the prices are:

> **One-year bond:** $\$P_{1t} = \frac{\$100}{1+i_{1t}}$

> **Two-year bond:** $\$P_{2t} = \frac{\$100}{(1+i_{1t})(1+i^e_{1,t+1})}$

> **Three-year bond:** $\$P_{3t} = \frac{\$100}{(1+i_{1t})(1+i^e_{1,t+1})(1+i^e_{1,t+2})}$

Where i_{1t} is the current one-year nominal interest rate and $i^e_{1,t+1}$, $i^e_{1,t+2}$ are the one-year rates financial markets expect next year and the year after.

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2. The Arbitrage Condition

Choosing between a one-year and a two-year bond

Suppose you must choose between a one-year bond and a two-year bond, and you care **only about the expected return** (risk neutral — the **expectations hypothesis**).

- **One-year bond:**
  - for each $1 invested, you get $(1 + i_{1t})$ dollars next year.
- **Two-year bond:**
  - its price is $\$P_{2t}$.
    - For every dollar invested you buy $\frac{1}{\$P_{2t}}$ bonds today.
  - After one year, the bond has one year left to maturity → it is now a one-year bond.
    - You can sell it at the expected price $\$P^e_{1,t+1}$.
    - So for every dollar invested, you expect to receive $\frac{\$P^e_{1,t+1}}{\$P_{2t}}$ dollars next year.

For both bonds to coexist in equilibrium, they must yield the same return. This gives the **arbitrage condition**:

$$1 + i_{1t} = \frac{\$P^e_{1,t+1}}{\$P_{2t}}$$

If this did not hold, say the one-year bond yielded less, no one would hold it, and the market would not be in equilibrium.

Rearranging:

> $\$P_{2t} = \frac{\$P^e_{1,t+1}}{1+i_{1t}}$

This says: **the price of a two-year bond today is the present value of the expected price of a one-year bond next year.**

Substituting for $\$P^e_{1,t+1} = \frac{\$100}{1+i^e_{1,t+1}}$, we recover:

$$\$P_{2t} = \frac{\$100}{(1+i_{1t})(1+i^e_{1,t+1})}$$

This confirms: arbitrage between bonds of different maturities implies that bond prices equal the present value of their payments, discounted using current and expected future short-term rates. The same logic extends to any maturity:

$$\$P_{3t} = \frac{\$P^e_{2,t+1}}{1+i_{1t}} = \frac{\$P^e_{1,t+2}}{(1+i_{1t})(1+i^e_{1,t+1})}$$

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3. Yield to Maturity

The **yield to maturity** on an n-year bond is the **constant annual interest rate that makes the present value of future payments equal to the bond's current price**:

$$\$P_{nt} = \frac{\$100}{(1+i_{nt})^n}$$

For a two-year bond: $\$P_{2t} = \frac{\$100}{(1+i_{2t})^2}$.

Equating this with the price derived from arbitrage:

$$\frac{\$100}{(1+i_{2t})^2} = \frac{\$100}{(1+i_{1t})(1+i^e_{1,t+1})}$$

$$(1+i_{2t})^2 = (1+i_{1t})(1+i^e_{1,t+1})$$

Using the small-number approximation:

> $$i_{2t} \approx \frac{1}{2}(i_{1t} + i^e_{1,t+1})$$

The two-year interest rate is approximately the average of the current one-year rate and next year's expected one-year rate.

Generalising to n years:

> $i_{nt} = \frac{1}{n}(i_{1t} + i^e_{1,t+1} + \cdots + i^e_{1,t+n-1})$

This is the **expectations hypothesis: long-term interest rates reflect the average of current and expected future short-term rates.**

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4. Now We Introduce Risk

Up to here, investors were assumed to be risk-neutral. In reality, they care about risk — and holding a two-year bond for one year is risky, because the price at which you will sell it next year is uncertain (*price risk*).

To compensate, investors require a **risk premium** x. The arbitrage condition becomes:

$$1 + i_{1t} + x = \frac{\$P^e_{1,t+1}}{\$P_{2t}}$$

Rearranging:

$$\$P_{2t} = \frac{\$P^e_{1,t+1}}{1 + i_{1t} + x} = \frac{\$100}{(1 + i_{1t} + x)(1 + i^e_{1,t+1})}$$

Going from prices to yields and approximating:

> $i_{2t} \approx \frac{1}{2}(i_{1t} + i^e_{1,t+1} + x)$

The two-year rate is the average of the current and expected one-year rate, **plus** the risk premium. Since price risk increases with maturity, x typically increases with maturity, reaching 1–2% for long-term bonds. The generalised formula is:

> $i_{nt} = \frac{1}{n}(i_{1t} + i^e_{1,t+1} + \cdots + i^e_{1,t+n-1} + x)$

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5. The Yield Curve

- The **yield curve** (also called the *term structure of interest rates*) is the graphical representation of the relationship between yields and maturities

- yield on the vertical axis, maturity (in years) on the horizontal axis.

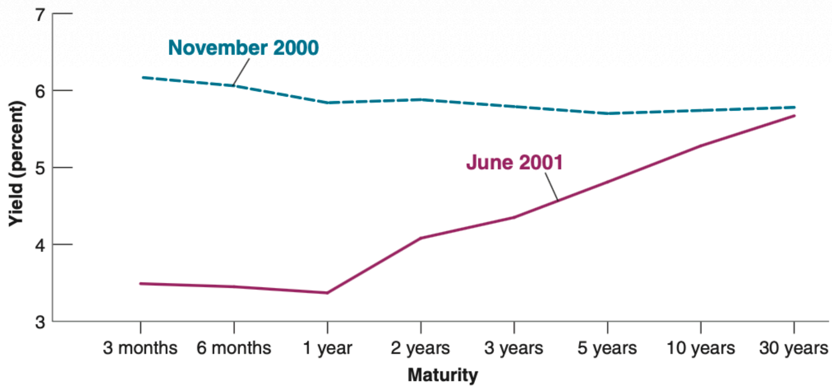

Figure 15.2
US yield curves: 1 November 2000 and 1 June 2001
The yield curve, which was slightly downward sloping on November 1, 2000, was sharply upward sloping seven months later.
Source: FRED. Series DGS1MO, DGS3MO, DGS6MO, DGS1, DGS2, DGS3, DGS5, DGS7, DGS10, DGS20, DGS30.

Reading the slope (without risk)

Starting from $i_{2t} = \frac{1}{2}(i_{1t} + i^e_{1,t+1})$, we can derive:

$$i_{2t} - i_{1t} = \frac{1}{2}(i^e_{1,t+1} - i_{1t})$$

This gives us a direct reading of market expectations:

- **Positive slope** $(i_{2t} > i_{1t})$
  - markets expect short-term rates to **rise** $(i^e_{1,t+1} > i_{1t})$
  - → signals expected **contractionary** monetary policy.
- **Negative slope** $(i_{2t} < i_{1t})$
  - markets expect short-term rates to **fall** $(i^e_{1,t+1} < i_{1t})$
  - → signals expected **expansionary** monetary policy.
- **Flat** $(i_{2t} = i_{1t})$ → markets expect **no change** in short-term rates $(i^e_{1,t+1} = i_{1t})$.

Once we account for risk

The **risk premium x increases with maturity**. Therefore:

- An **upward slope** does not necessarily imply expected higher future rates
  - It could just reflect the **rising risk premium**.
  - This is the *normal* shape of the yield curve.
- A **downward slope** is **rare**.

- It requires that the expected decrease in future short-term rates is strong enough to more than offset the upward push from the rising risk premium.

> **Reference:** Book p. 327 — "Interpreting the yield curve"

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6. The Price of Stocks

- **Stocks (or shares) are securities that represent a fraction of the ownership of a corporation.**
- Unlike bonds, they do not pay predetermined amounts; instead they pay **dividends**, determined by the firm's profits.

Setting up the arbitrage condition

Suppose you must choose between a one-year bond and holding a stock for one year.

- **One-year bond:** for every $1 invested, you receive $(1 + i_{1t})$ next year.
- **Stock for one year:**
  - $\$Q_t$ = **price of the stock today**
    - the **ex-dividend price** → after this year's dividend has been paid, so the first dividend you receive is next year's
  - $\$D^e_{t+1}$ **= expected dividend next year**
  - $\$Q^e_{t+1}$ **= expected stock price next year**
- For every dollar invested you buy $\frac{1}{\$Q_t}$ shares.
  - Next year you sell the stock and receive $\$D^e_{t+1} + \$Q^e_{t+1}$ per share.
  - Expected return per dollar: $\frac{\$D^e_{t+1}+\$Q^e_{t+1}}{\$Q_t}$.
- Holding a stock is much riskier than holding a one-year bond.
  - The risk premium for stocks is called the **equity premium** x.
  - **The arbitrage condition requires:**

$$\frac{\$D^e_{t+1} + \$Q^e_{t+1}}{\$Q_t} = 1 + i_{1t} + x$$

Rearranging:

$$\$Q_t = \frac{\$D^e_{t+1}}{1 + i_{1t} + x} + \frac{\$Q^e_{t+1}}{1 + i_{1t} + x}$$

The stock price today equals:

- **present value of the expected dividend next year**

  +

- **present value of the expected stock price next year.**

Extending forward

Substituting for $\$Q^e_{t+1}$ using the same arbitrage condition one period ahead, and repeating indefinitely:

- $Q^e_{t+1} = \frac{D^e_{t+2}+Q^e_{t+2}}{1+i^e_{1,t+1}+x}$
- We obtain the fundamental stock price formula:

  > $Q_t = \frac{D^e_{t+1}}{1+i_{1t}+x} + \frac{D^e_{t+2}}{(1+i_{1t}+x)(1+i^e_{1,t+1}+x)} + \cdots + \frac{D^e_{t+n}}{(1+i_{1t}+x)\cdots(1+i^e_{1,t+n-1}+x)} + \frac{Q^e_{t+n}}{(1+i_{1t}+x)\cdots(1+i^e_{1,t+n-1}+x)}$

- **last term (the present value of the price n periods out) goes to zero as** $n \to \infty$ (provided the discount rate is positive and prices do not explode).
  - to see why suppose interest rate is constant and equal to i
    - $\frac{Q^e_{t+n}}{(1+i_{t+x})(1+i_{t+1+x})\cdots(1+i_{t+n-1+x})} = \frac{Q^e_{t+n}}{(1+i+x)^n}$
      - the total discount factor (denominator) "explodes" as $n \to \infty$

> $\$Q_t = \frac{\$D^e_{t+1}}{(1+i_{1t}+x)} + \frac{\$D^e_{t+2}}{(1+i_{1t}+x)(1+i^e_{1,t+1}+x)} + \cdots + \frac{\$D^e_{t+n}}{(1+i_{1t}+x)\cdots(1+i^e_{1,t+n-1}+x)}$

In **real terms**, replacing nominal rates with real rates:

> $Q_t = \frac{D^e_{t+1}}{(1+r_{1t}+x)} + \frac{D^e_{t+2}}{(1+r_{1t}+x)(1+r^e_{1,t+1}+x)} + \cdots$

Three implications

This formula has three key implications:

- ↑ Expected future real dividends D^e → ↑ Q_t

- ↑ Current or expected future real interest rates r_{1t} or r^e → ↑ discount rate i → ↓ PV of dividends → ↓ Q_t

- ↑ Equity premium x → ↑ discount rate → ↓ Q_t

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7. Policies Announced and Implemented

An important distinction for understanding stock market reactions:

Situation	Effect on Q_t
Policy **directly implemented**, **fully anticipated**	No reaction, already priced in
Policy **directly implemented**, **partly unexpected**	Q_t reacts (partially)
Policy **announced** and **believed**	Q_t reacts immediately
Policy **announced** but **not believed**	No reaction (Q_t unchanged)

> The key principle: stock prices react to **new information**, not to events that have already been priced in.

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8. Stock Prices and the IS-LM Model

Case A: Monetary expansion (standard IS-LM)

The central bank lowers the policy rate → LM shifts down from LM to LM'.

- If **fully anticipated:** no reaction (Q_t unchanged).
- If **at least partly unexpected:**
  - $r \downarrow$ → discount rate ↓ → PV of expected dividends ↑ → Q_t ↑
  - $y \uparrow$ → profits ↑ → expected dividends D^e ↑ → Q_t ↑

- **Overall:** $Q_t \uparrow$

Case B: Increase in consumer confidence / expansionary fiscal policy (standard IS-LM)

IS shifts right. What happens depends on what investors expect the central bank to do:

1. CB does not intervene (keeps r constant): $y \uparrow$ → profits $\uparrow$ → $D^e \uparrow$ → $Q_t \uparrow$
2. CB partially intervenes (raises r but less than enough to keep y constant): both $r \uparrow$ and $y \uparrow$ → Q_t **ambiguous**
3. CB fully counteracts (raises r enough to keep y at y_n): y unchanged, $r \uparrow$ → PV of dividends $\downarrow$ → $Q_t \downarrow$

Case B continued: non-standard IS-LM (LM upward sloping)

When IS shifts right and LM is upward sloping, both y and r increase. The net effect on Q_t depends on the **slope of LM**:

- **Flat LM:** large Δy, small Δr → dividend effect dominates → $Q_t \uparrow$
- **Steep LM:** small Δy, large Δr → discount rate effect dominates → $Q_t \downarrow$

If we then consider how the CB reacts:

1. CB increases money supply to stabilise r → r unchanged, $y \uparrow$ → $Q_t \uparrow$
2. CB does not intervene → Q_t **ambiguous**
3. CB fears inflation and contracts (LM shifts up to LM''): y unchanged or falls, $r \uparrow\uparrow$ → $Q_t \downarrow$

> **General lesson:**
>
> - Stock prices depend on current and future movements in activity, BUT there is no simple relation between stock prices and output.
> - The **direction of the response depends on**
>   1. what markets expected in the first place
>   2. the source of the shock
>   3. expected central bank behaviour

9. Risk, Bubbles, Fads and Asset Pricing

Not all movements in stock prices come from news about fundamentals (dividends or interest rates). Two additional sources of movement:

Changes in the equity premium

- The equity premium x is not constant
  - After the Great Depression it was very high (around 7%), declining to below 4% today.
  - Part of the 2008 stock market crash reflected a sharp increase in perceived risk and uncertainty (a jump in x) not just lower expected dividends.

Bubbles

The ***fundamental value*** of a stock is the present value of expected future real dividends given in the formula above. **Stocks can deviate from this value**.

> A **rational speculative bubble** occurs when an **asset price exceeds its fundamental value because current owners believe they can resell it at an even higher price in the future.**
> Stock prices increase simply because investors *expect* them to increase, even though fundamentals do not justify it.

When prices are subject to bubbles, the **last term in the stock price formula (the present value of the future price) does *not* go to zero**, because people expect large future price increases. The stock price is then **no longer equal to the present value of expected dividends.**

Historical examples: Black October 1929 (US market fell 23% in two days), October 1987 (Dow Jones fell 22.6% in a single day), the Nikkei index (rose from ~13,000 in 1985 to ~35,000 in 1989, then fell to 16,000 in 1992), tulipmania in 17th-century Holland, the US housing bubble in the 2000s.

Fads

Fads are cases where **stocks become high-priced due to excessive optimism, simply because their prices have risen in the past,** not because investors expect to resell at even higher prices. Investors extrapolate from past returns, ignoring fundamentals.

> **Garber [2000] on bubbles:** *"Bubble is one of the most beautiful concepts in economics in that it is a fuzzy word lacking a solid operational definition. The definition most often used is that part of asset price movement that is unexplainable based on fundamentals."*

An important note on the housing market

The same logic applies to house prices: absent bubbles, **house prices depend on current and expected future interest rates, current and expected future rents, and a risk premium** (analogous to dividends for stocks).

The US housing price-to-rent ratio rose ~60% from the late 1990s to its 2006 peak, a rise that proved difficult to interpret in real time (fundamentals vs. bubble debate), but was widely recognised as a bubble in retrospect.

> **Suggested reading:** *Manias, Panics, and Crashes* (Kindleberger & Aliber); B. Malkiel, *A Random Walk Down Wall Street*

> **Optional – Yield Curve and Recessions:** In the US, the yield curve has historically inverted 5–16 months (average ~12 months) before a recession. An **inverted yield curve** (short-term rates > long-term rates, negative term spread) signals that markets expect future interest rates to fall — consistent with expectations of economic slowdown and future monetary easing.

Chapter 16 - Expectations, consumptions and investments

☑ Reviewed ☐

📊 Macroeconomics: Chapter 16 – Expectations, Consumption and Investment

Reference: Lectures 18, 19, 20 (25–27 March 2025) and Chapter 16 Textbook (pp. 341–357)

Context: Extensions – Expectations. We now examine how expectations shape the two main components of spending: consumption and investment.

> 💡 **Key message:** Both consumption and investment decisions depend very much on expectations about the future — not just on current income or current interest rates.

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1. Consumption

The Traditional Model and Its Limits

The basic IS-LM model assumed:

$$C = c_0 + c_1(Y - T)$$

This is not fully realistic. Consumption clearly depends on more than just current disposable income.

The Modern Theory of Consumption

Around the 1950s, two economists independently developed a richer theory:

- **Milton Friedman** → the ***Permanent Income Theory of Consumption***: consumers look *beyond* current income and base decisions on their *permanent* (long-run average) income.

- **Franco Modigliani** → the ***Life-Cycle Theory of Consumption***: consumers' natural planning horizon is their **entire lifetime**.

Both agree on a core insight: when deciding how much to consume, people consider their **total wealth**, not just this year's income.

Components of Wealth

Total wealth is divided into two broad categories:

1. **Non-Human Wealth**
    - **Financial wealth**: value of savings accounts, stocks, and bonds owned.
    - **Housing wealth**: value of house(s) owned minus any mortgage still due.
2. **Human Wealth**
    - The **present value** of current and expected future **after-tax labour income** over one's working life.

> **Total Wealth = Non-Human Wealth + Human Wealth**

The Consumption Function

The very foresighted consumer sets consumption as a proportion of total wealth, maintaining roughly constant consumption over their lifetime (**consumption smoothing**):

$$C_t = C(\text{Total Wealth}_t)$$

In practice, because of the reasons below (liquidity constraints, precautionary saving, etc.), consumption also depends on current after-tax labour income:

$$C_t = C\left(\underbrace{Y_{Lt} - T_t}_{\text{current inc.}}, \underbrace{W_t}_{\text{total wealth}}\right) \quad (+,\ +)$$

How Expectations Affect Consumption

Expectations enter in **two channels**:

- **Directly** through **human wealth**:
    - consumers form **expectations about future labour income, real interest rates, and taxes.**
- **Indirectly** through **non-human wealth** (stocks, bonds, housing):

- **asset prices already incorporate market expectations about future dividends and interest rates.**

Two important implications:

- Consumption is likely to respond **less than one-for-one** to fluctuations in current income
  - the more transitory the income change, the smaller the consumption response
- Consumption **may move even if current income does not change**
  - e.g., an **optimistic election** result leads to higher expected future income → $C_t \uparrow$ immediately.

Empirical Evidence

- C is sensitive to **current income** changes
  - **especially when future expectations are negative**.

- The **marginal propensity to consume** is higher when the change in income is perceived as **permanent** (≈ 1) than when it is **temporary** (< 1).

- Consumption responds **less than one-for-one** to temporary changes.

- Consumption smoothing over a lifetime is **rarely applied fully** in practice.

- People are **less forward-looking than the theory implies, especially when liquidity-constrained.**

Exercise 7 – Events Affecting C_t

Event	Chain of effects	Effect on C_t
Demand for houses $\downarrow$	House prices $\downarrow$ → Housing wealth $\downarrow$ → Total wealth $\downarrow$	$C_t \downarrow$
Stock market drop	Financial wealth $\downarrow$ → Total wealth $\downarrow$	$C_t \downarrow$
Permanent $\uparrow$ in taxes T	Human wealth $\downarrow$ → Total wealth $\downarrow$	$C_t \downarrow$

Event	Chain of effects	Effect on C_t
Expected ↑ in nominal interest rate	Discount rate ↑ → PV of wealth ↓	C_t ↓

2. The Intertemporal Budget Constraint

Setup (Exercise 8 – EB Ch. 4, pp. 118–121)

We formalise intertemporal consumption choice using a two-period model:

- $y_t - T_t$: disposable labour income in period t
- $y^e_{t+1} - T^e_{t+1}$: expected disposable labour income in period $t+1$
- Non-human wealth (financial + housing) = 0
- r: real interest rate at which the individual can borrow or lend

Step 1 — Budget constraint at time t:

$$y_t - T_t = C_t + S$$

where S is saving: positive if the person lends ($C_t < y_t - T_t$), negative if the person borrows ($C_t > y_t - T_t$), and zero if they spend exactly their income.

Step 2 — Budget constraint at $t+1$:

$$y^e_{t+1} - T^e_{t+1} + (1+r)S = C_{t+1}$$

At $t+1$ there is no period-3, so all resources are consumed.

Step 3 — The Intertemporal Budget Constraint (IBC):

From Step 1: $S = (y_t - T_t) - C_t$. Substituting into Step 2 and rearranging:

> $C_t + \frac{C_{t+1}}{1+r} = (y_t - T_t) + \frac{y^e_{t+1} - T^e_{t+1}}{1+r}$

- **Left side:** present discounted value of consumption.
- **Right side:** present discounted value of **human wealth** (after-tax labour income).

Equivalently, written as a line in (C_t, C_{t+1}) space:

$$C_{t+1} = \left[(1+r)(y_t - T_t) + (y^e_{t+1} - T^e_{t+1})\right] - (1+r)C_t$$

Key features of the IBC line:

- **Slope:** $-(1+r)$
- **Vertical intercept** (if $C_t = 0$): $(1+r)(y_t - T_t) + (y^e_{t+1} - T^e_{t+1})$
- **Horizontal intercept** (if $C_{t+1} = 0$): $(y_t - T_t) + \frac{y^e_{t+1} - T^e_{t+1}}{1+r}$
- **Endowment point A:** the IBC always passes through $(y_t - T_t,\ y^e_{t+1} - T^e_{t+1})$ — the outcome if the person spends exactly their income in each period.

Equilibrium

The optimum (point E) is the **tangency between the IBC and the highest attainable indifference curve**. Depending on where E falls:

- $C^*_t > (y_t - T_t)$ → person is a **borrower** (uses credit today, repays at $t+1$)
- $C^*_t < (y_t - T_t)$ → person is a **lender** (saves today, consumes more at $t+1$)
- $C^*_t = (y_t - T_t)$ → person consumes exactly their income in each period

Income and Interest Rate Effects

When C_t and C_{t+1} are **normal goods** (consumption rises with income):

Shock	Effect on IBC	Effect on C^*_t
Transitory ↑ in $(y_t - T_t)$ only	Parallel rightward shift	C^*_t ↑ (small)
↑ in expected future income $(y^e_{t+1} - T^e_{t+1})$ only	Upward parallel shift	C^*_t ↑ and C^*_{t+1} ↑
Permanent ↑ (both periods)	Large rightward + upward shift	C^*_t ↑↑ (larger effect)
r ↑ for borrowers	Debt more expensive	C_t ↓ (demand for debt ↓)

Adding Non-Human Wealth

If at time t individuals hold positive non-human wealth $W_{FH,t}$ (financial + housing), the IBC expands to:

$$C_t + \frac{C_{t+1}}{1+r} = (y_t - T_t) + \frac{y^e_{t+1} - T^e_{t+1}}{1+r} + W_{FH,t}$$

This confirms the general consumption function: $C_t = C(\text{Total Wealth}_t)$.

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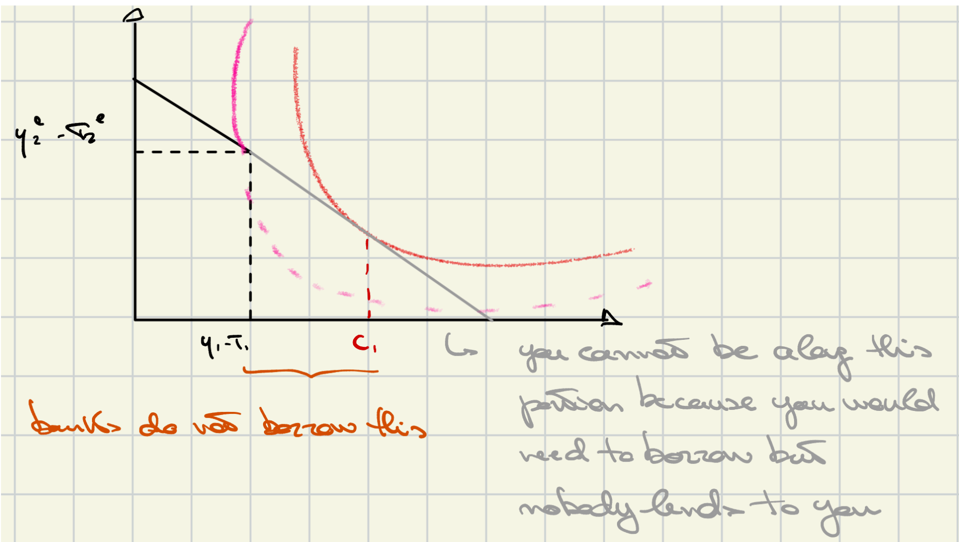
3. Liquidity-Constrained People

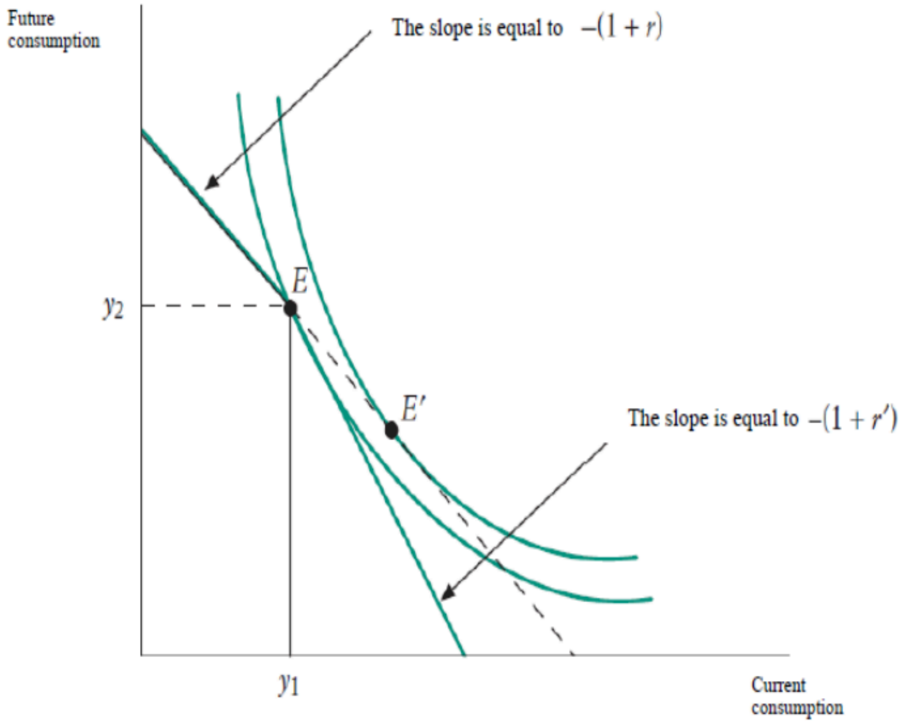
Definition

A **liquidity-constrained (LC)** person is someone who consumes **less than they would optimally like to** because they cannot access credit at a reasonable rate.

- Two reasons:
  1. They cannot obtain credit at all (no access to borrowing).
  2. The interest rate on borrowing is too high, so they choose not to borrow.

- Their **IBC is truncated**
  - they cannot move into the borrowing region ($C_t > y_t - T_t$).
  - Their equilibrium is therefore the endowment point $E = A$, where they spend exactly their income each period.

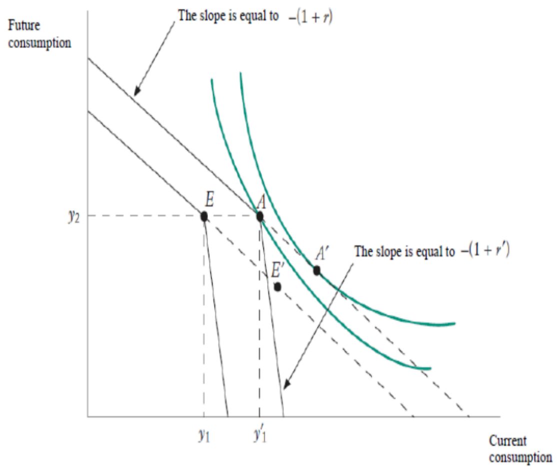


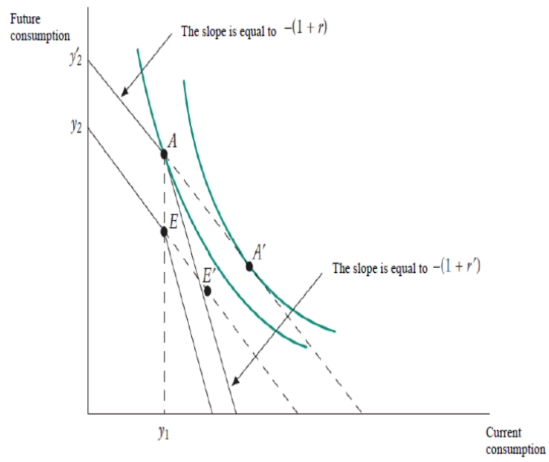


- The dashed line represent the same situation as before.
- The greenish line concerns the case where the current interest rate is too high
  - Liability-constrained individuals may face higher borrowing rates, therefore borrowing is too costly and they may stay closer to the endowment point E or A (look also at graph below)
    - So the new optimum is found by looking for the highest indifference curve reachable under the green line.

Behaviour Compared to Non-Constrained Individuals

- Current income increases
- Expected future income increases





Scenario	Non-LC individual	LC individual
Current income ↑ (temporary)	Moves from E' to A'; saves some of the increase	Moves from E to A; **consumes the full increase** → to get closer to the desired levelo of consumption in t_1
Expected future income ↑	$C_t \uparrow$ and $C_{t+1} \uparrow$	C_t **unchanged today**; reacts only in period 2 when income actually rises → not able to increase C today

Key insight: LC individuals **react a lot** to increases in *current* income (they spend the whole increase at once to get closer to their desired level), and **do not react at all** to changes in *expected future* income (they have **no access to credit** to shift future income to the present).

Policy Implications (Announcements)

Consider a government announcement of a **future tax cut at** $t+1$**:**

- If the announcement is **not believed**: **nothing happens**.
- If **fully anticipated** (already expected): **nothing happens**.
- If **believed and at least partly unexpected**:
  - **LC individuals**: cannot react because they have no access to credit today → C_t unchanged.
  - **Non-LC individuals**: react immediately when there is the announcement → $C_t \uparrow$.

If many individuals in the economy are liquidity-constrained, a fiscal policy acting through expected future income will have **a smaller aggregate consumption effect**.

4. Investment

From the Basic Model to Expectations

- In Chapter 5 we assumed $I = I(Y, r)$.
- However, investment decisions depend fundamentally on **expectations about the future:**
    - expected profits, expected interest rates, and the lifespan of the capital goods being purchased

The Investment Decision

A firm deciding whether to buy a new machine must compare:

- The **present value of expected profits** from having the machine: $V(\Pi^e)$
- The **cost** of the machine (normalised to 1 in real terms)

Decision rules:

- $V(\Pi^e) > 1$ → **invest**
- $V(\Pi^e) < 1$ → **do not invest**
- $V(\Pi^e) = 1$ → indifferent (look for something more profitable)

Depreciation

- Every year, a machine loses a **fraction δ (delta)** of its value/productivity
    - **depreciation rate**
    - After one year only $(1 - \delta)$ of the machine remains productive
        - after two years, $(1 - \delta)^2$; and so on
    - Typical values range from 2.5% for office buildings to 15% for communication equipment.

The firm buys the machine at time t but it becomes operational only at $t + 1$ (installation lag).

The Present Value of Expected Profits

$$V(\Pi_t^e) = \frac{\Pi_{t+1}^e}{1+r_t} + \frac{(1-\delta)\,\Pi_{t+2}^e}{(1+r_t)(1+r_{t+1}^e)} + \frac{(1-\delta)^2\,\Pi_{t+3}^e}{(1+r_t)(1+r_{t+1}^e)(1+r_{t+2}^e)} + \cdots$$

- Each term:
  - expected real profits at that date, adjusted for accumulated depreciation, discounted back to time t using current and expected future **real** interest rates.
- The **higher expected profits** → **higher** $V(\Pi^e)$ → **more investment**.
- The **higher expected interest rates** → **lower** $V(\Pi^e)$ → **less investment**.

Aggregate Investment Function

Letting I_t denote aggregate investment and Π_t denote profits per unit of capital for the economy as a whole:

$$I_t = I[V(\Pi_t^e)] \quad (+)$$

A convenient special case → **static expectations:**

- if firms expect future profits and interest rates to remain at today's level ($\Pi_{t+k}^e = \Pi_t$, $r_{t+k}^e = r_t$ for all k), this simplifies to:

$$V(\Pi_t^e) = \frac{\Pi_t}{r_t+\delta}$$

- → geometric series
  - $V = \frac{\Pi_t}{1+r}\left[1 + \frac{1-\delta}{1+r} + \left(\frac{1-\delta}{1+r}\right)^2 + \cdots\right]$
  - $1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$
  - with $a = \frac{1-\delta}{1+r}$
  - and then $V = \frac{\Pi_t}{1+r} \cdot \frac{1}{1-\frac{1-\delta}{1+r}} = \frac{\Pi_t}{r+\delta}$

And investment becomes:

$$I_t = I\left(\frac{\Pi_t}{r_t+\delta}\right)$$

where $(r_t + \delta)$ is the **user cost (rental cost) of capital,** the cost to the firm of using one unit of capital for a year, combining the opportunity cost of funds (r_t) and physical depreciation (δ).

Current Profit vs. Expected Profit

Theory says investment should be primarily forward-looking and depend on *expected future* profits. Yet empirically, **current profit strongly predicts investment**. The reason parallels the LC story for consumers:

- A firm with **low current profit** must borrow to invest
  - but may be **reluctant to take on debt** given downside risk.
- A firm with **high current profit** can **self-finance** via retained earnings
  - avoiding banks and proceeding with investment directly.

→ **Banks may refuse to lend to firms with low current profit even if future prospects look good**.

The empirically richer investment function is therefore:

$$I_t = I[V(\Pi_t^e),\ \Pi_t] \quad (+,\ +)$$

Profit and Output

What determines profit per unit of capital Π_t?

→ Primarily **the level of output relative to the existing capital stock**:

$$\Pi_t = \Pi\left(\frac{Y_t}{K_t}\right) \quad (+)$$

- For a given K_t: higher output Y_t → higher profit per unit of capital.
- For a given Y_t: higher capital stock K_t → lower profit per unit of capital.

This links current and expected *output* to investment:

- Current output Y_t → current profit Π_t → investment I_t
- Expected future output → expected future profits Π^e → $V(\Pi_t^e)$ → investment I_t

So ultimately:

$$I_t = I[V(Y_t^e),\ Y_t]$$

Exercise 9 – Stock Prices and Investment (IS-LM with non-standard LM)

T/F: If $\bar{I}$ increases from t onwards, stock prices Q_t will definitely rise.

Answer: FALSE. Two effects work in opposite directions:

- **Dividend effect:** $\bar{I} \uparrow$ → IS shifts right → $Y \uparrow$ → profits $\uparrow$ → $D^e \uparrow$ → $Q_t \uparrow$
- **Discount rate effect:** $r \uparrow$ → discount rate $\uparrow$ → PV of dividends $\downarrow$ → $Q_t \downarrow$

The net effect **depends on the slope of the LM curve** (which determines how much r rises relative to Y). No unambiguous conclusion without knowing the LM slope.

---

5. The Volatility of Consumption and Investment

Feature	Consumption	Investment
Sensitivity to transitory shocks	**Low** consumption smoothing	**Low** firms understand temporary booms (e.g. Christmas)
Response to permanent shocks	At most one-for-one increase	Can **exceed** the increase in sales (capital stock must adjust)
Volatility	Low (range ≈ −5% to +3%)	High (range ≈ −29% to +24%)
Share of GDP	~70%	~15%
Contribution to output fluctuations	Roughly equal to investment	Roughly equal to consumption

- **Why is investment more volatile?**
  - If sales increase *permanently*, a firm with capital-to-sales ratio of 3 must spend 3× the increase in sales on new capital right away
    - leading to a large but short-lived spike in investment spending.
    - consumption has no equivalent accelerator mechanism.

> **Bottom line:** investment is much more volatile than consumption, but because it is only ~15% of GDP, absolute fluctuations in investment and

> consumption are of roughly equal magnitude — both matter for output dynamics.

🗂️ Recap Table

Concept	Key formula	What it says
Consumption function	$C_t = C(\text{Total Wealth}_t)$	C depends on human + non-human wealth
Intertemporal Budget Constraint	$C_t + \frac{C_{t+1}}{1+r} = (y_t - T_t) + \frac{y^e_{t+1} - T^e_{t+1}}{1+r}$	PDV of consumption = PDV of human wealth
IBC with non-human wealth	$+W_{FH,t}$ added to right side	Financial + housing wealth shifts the IBC out
Investment decision	Compare $V(\Pi^e_t)$ with cost (= 1)	Invest if and only if PV of expected profits > cost
Present value of profits	$V(\Pi^e_t) = \frac{\Pi^e_{t+1}}{1+r_t} + \frac{(1-\delta)\Pi^e_{t+2}}{(1+r_t)(1+r^e_{t+1})} + \cdots$	Discounted stream of profits net of depreciation
Static expectations special case	$V(\Pi^e_t) = \frac{\Pi_t}{r_t + \delta}$	User cost = real rate + depreciation rate
Richer investment function	$I_t = I[V(\Pi^e_t), \Pi_t]$	Both expected and current profits matter
Profit and output	$\Pi_t = \Pi(Y_t / K_t)$	Profits rise with output, fall with capital stock

Chapter 17 - Expectations, output and policy

Reviewed	☐

Reference: Lectures 20, 21, 22 Blanchard Ch. 17 (pp. 363–369) · Ferraguto Ch. 4 (pp. 126–137)

Context: Extensions – Expectations. Building on Chapters 15 and 16, we now integrate expectations of the future directly into the IS-LM model and study how monetary and fiscal policy work when markets look ahead.

> 💡 **Key message:** The effects of monetary and fiscal policy depend very much on whether and how those policies affect expectations of future income and future interest rates. If expectations do not react, policy effects on output are small.

---

1. IS — The Expectations-Augmented IS Curve

Starting Point: The Chapter 6 IS Relation

Recall the goods-market equilibrium from Chapter 6:

$$Y = C(Y - T) + I(Y, r + x) + G$$

where r is the policy rate set by the central bank and x is the risk premium, so $r + x$ is the actual borrowing rate faced by firms.

Define **aggregate private spending** A as the sum of consumption and investment:

$$A(Y, T, r, x) \equiv C(Y - T) + I(Y, r + x)$$

The IS relation then becomes:

$$Y = A(Y, T, r, x) + G$$

The signs of each argument are $(+, -, -, -)$:

Variable ↑	Economic effect	Direction
Output Y	Both C and I increase	$A \uparrow$
Taxes T	Disposable income falls → $C \downarrow$	$A \downarrow$
Policy rate r	Investment more expensive → $I \downarrow$	$A \downarrow$
Risk premium x	Borrowing costs rise → $I \downarrow$	$A \downarrow$

Introducing Expectations

We now assume x is constant and ignore it. The key extension is allowing spending to depend on **expected future** values of output, taxes, and the interest rate, in addition to their current values. The IS becomes:

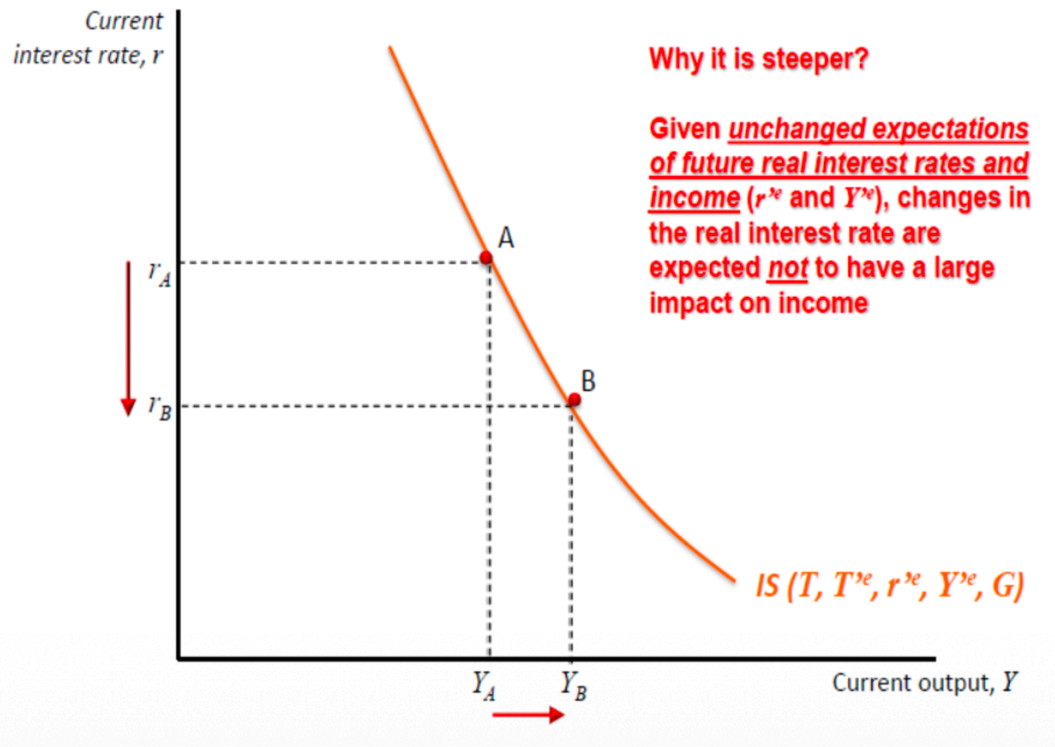
$$Y = A(Y,\, T,\, r,\, Y'^e,\, T'^e,\, r'^e) + G$$

where primes denote future-period values and superscript e denotes expectations. This is the **expectations-augmented IS curve**. Signs: $(+, -, -, +, -, -)$.

Why each sign?

- $Y'^e \uparrow$ → consumers feel wealthier (higher human wealth) → $C \uparrow$; firms expect higher future profits → $I \uparrow$
- $T'^e \uparrow$ → expected future disposable income falls → $C \downarrow$
- $r'^e \uparrow$ → present value of expected future profits falls → $I \downarrow$

Shape and Sensitivity



- explicit formula (solving for Y):

$$Y = \frac{1}{1-(c_1+d_1)}\left[c_0 - c_1 T + I_0 + G + c_2(Y'^e - T'^e) + d_2 Y'^e - d_4 r'^e - d_3 r\right]$$

from: $Y = [c_0 + c_1(Y - T) + c_2(Y'^e - T'^e)] + [I_0 + d_1 Y + d_2 Y'^e - d_3 r - d_4 r'^e] + G$

- The expectations-augmented IS curve is downward sloping but **steeper than the standard IS.** The intuition is twofold:
  1. **Small direct effect of r on spending**
     - A decrease in only the *current* interest rate, with **expected future rates unchanged**, barely changes the present value of profits or human wealth.
     - The **borrowing cost over the entire horizon moves very little**.
  2. **Small multiplier**

- A **change in current income**, given that **expected future income stays put**, has limited effect on consumption and investment.
    - **Consumers and firms understand the change is likely temporary**.

The IS shifts to the **right** when:

- $G \uparrow, T \downarrow$ (standard fiscal expansion)
- $Y'^e \uparrow$, $T'^e \downarrow$, $r'^e \downarrow$ (improved expectations)

---

2. LM — The Liquidity-Preference / Money-Supply Equation

- Expectations do **not** significantly affect how much money people wish to hold today.
    - Future expectations may affect consumption and investment but they do not significantly change how much liquidity people need right now
- Money demand depends on
    a. **current transactions**
    b. **current opportunity cost of holding money**

    → it is **myopic**. (it is short-sighted)

    - This means the **LM curve is unchanged by the introduction of expectations**.
    - Money demand depends only on current variables

Standard (flat) LM — central bank sets the interest rate:

$$r = \bar{r}$$

Non-standard LM — central bank controls the money supply:

$$\frac{M}{P} = Y \cdot L(r)$$

- In the non-standard version the LM is **upward sloping**:
    - if $Y \uparrow$ then money demand $M^d \uparrow$
    - with a fixed money supply, the interest rate must rise to restore equilibrium (bonds become more attractive at higher r, so money

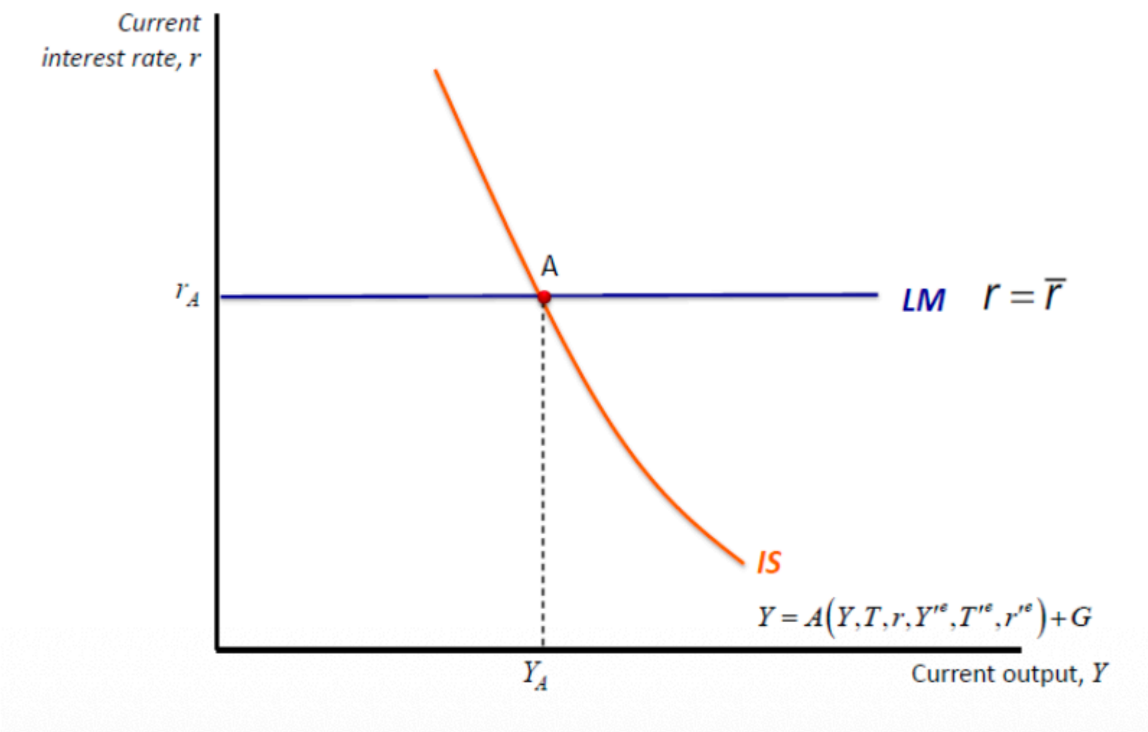
demand falls back).

3. Expectations-Augmented IS-LM: Standard Model

Combining the expectations-augmented IS with the flat LM gives equilibrium:

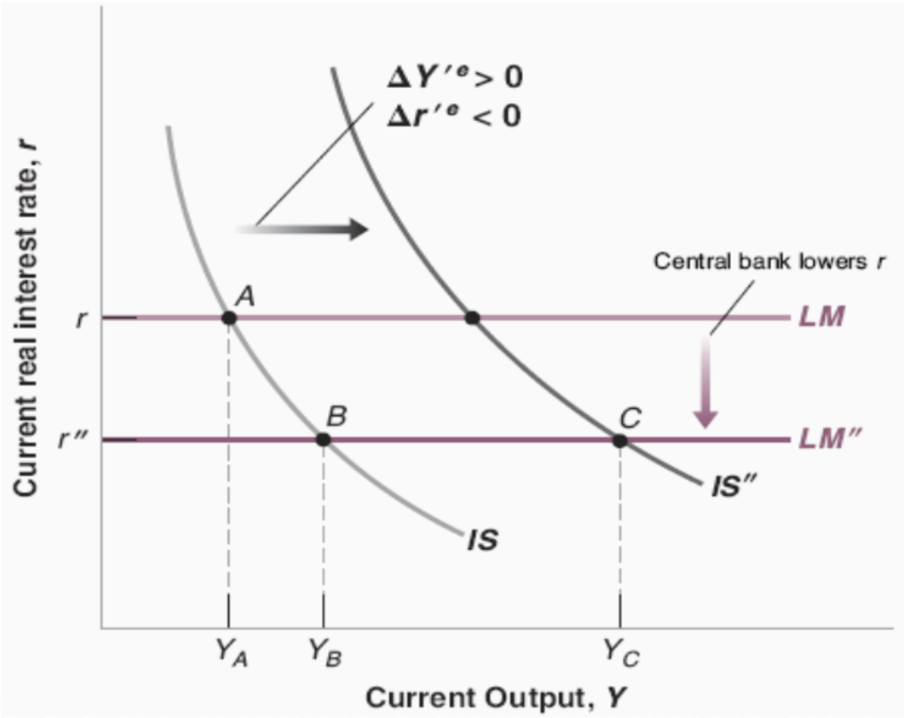
Relation	Equation
IS	$Y = A(Y, T, r, Y'^e, T'^e, r'^e) + G$
LM	$r = \bar{r}$

- The equilibrium is at the intersection:
  - the flat LM determines r, and the steep IS determines Y.
- The IS is very steep, so the position of equilibrium output depends heavily on expectations, not just on r.



4. Reasoning on the Standard Model

Consider an economy in **recession (IS shifts down)** (the initial IS is not in the graph, we hae the one already shifted). The central bank intervenes by lowering r.

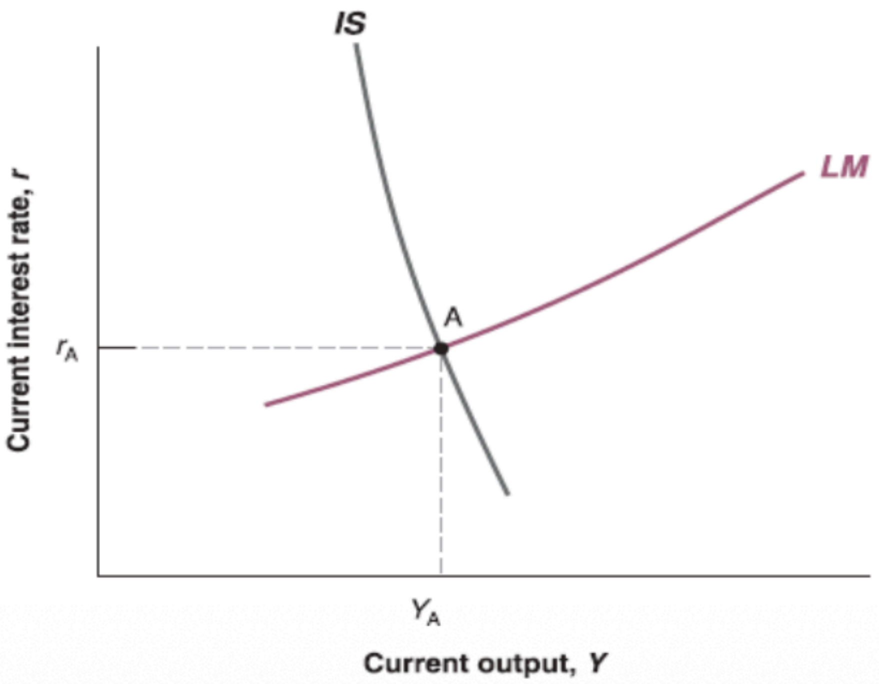


- **Scenario A → B (expectations unchanged):**
  - LM shifts down
  - Because the **IS is steep**, the **resulting increase in Y is *very small.***
  - A change in the current rate alone, if it does not alter what financial markets expect about the future, has **almost no effect**.
- **Scenario A → C (expectations react):**
  - if **financial markets believe the intervention signals persistently lower future interest rates and higher future output**, the **IS curve shifts to the right**.
  - The economy moves to point C, achieving a ***much larger* increase in Y.**

> 💡 **Key takeaway: Expectations are crucial.**
>
> - If the **policy is anticipated and believed → expectations are already adjusted.**
>   - when the policy is effectively introduced there is no actual change in IS because it had already shifted prior to the implementation of the policy (which was anticipated)

> - If the central bank can manage expectations (convincing markets about the future path of rates) the effect on output can be large even with a modest change in the current rate.

5. Non-Standard Expectations-Augmented IS-LM Model

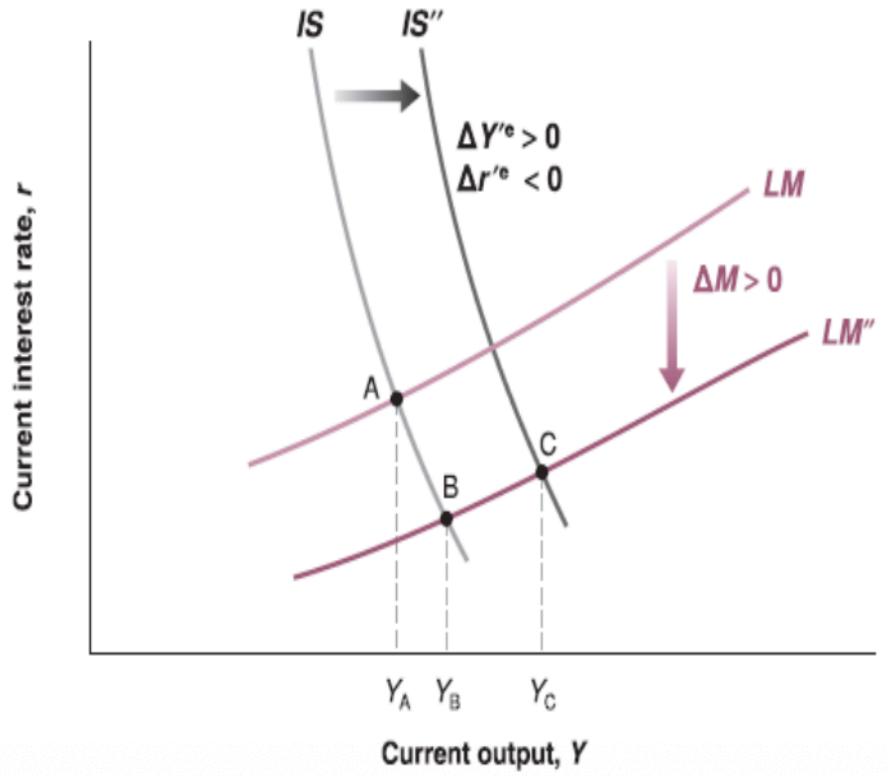


The expectations-augmented IS is the same as above. The LM returns to its upward-sloping form:

> $\frac{M}{P} = Y \cdot L(r)$

In this model, the central bank controls M rather than r. The equilibrium is again at the IS-LM intersection, but now the LM is positively sloped, so both r and Y are endogenously determined.

6. Reasoning on the Non-Standard Model



Consider the same **recessionary scenario**. The CB increases M^s → LM shifts down to LM'.
(remember LM: $i = -\frac{1}{f_2}\frac{\overline{M}}{P} + \frac{f_1}{f_2}Y$)

- **Case 1 — no change in expectations (A → B):**
    - the **effect on Y is small**, for the same reason: the steep IS limits the output response.
- **Case 2 — expectations react (A → C):**
    - financial markets **expect lower future r and higher future Y.** The IS shifts right. The economy ends up at point C with a significantly **larger increase in Y.**

> The **conclusion is identical to the standard model**:
>
> - **the effects of monetary policy depend on its impact on expectations.**
>   - Unchanged expectations → limited effect.
>   - Responsive expectations → large effect.

Effect on the Yield Curve

Throughout the chapter **we assume** $\pi_t = \pi^e_{t+1} = 0$, so nominal and real interest rates coincide ($i = r$). With only one-year and two-year bonds, the two-year yield equals the average of the current and expected future one-year yields:

$$i_{2t} = \frac{1}{2}\left(i_{1t} + i^e_{1,t+1}\right)$$

All yield-curve conclusions below use this formula.

- A **positively sloped** yield curve means $i_{2t} > i_{1t}$, i.e. $i^e_{1,t+1} > i_{1t}$.
- A **negatively sloped** curve means $i_{2t} < i_{1t}$.

---

7. Investors, Risk, and Country Comparisons

- A positive yield curve does **not** necessarily imply that financial markets expect future short-term rates to rise.
- The positive slope might also reflect a **risk premium** x that increases with maturity (**investors demand extra compensation for locking in funds over longer horizons**).

If country B's yield curve is steeper than country A's, **two explanations are equally valid**:

1. **Financial markets expect $i_{1,t+1}$ to rise *more* in B.**
2. Country **B carries a higher risk premium at longer maturities**.
    - The 2-year yield formula $i_{2t} = \frac{1}{2}(i_{1t} + i^e_{1,t+1}) + x$ shows that the **slope is determined by both the expected future interest rate and the risk premium** x
        - **allowing for a positive slope even if rates are not expected to rise.**

We **cannot** infer from a positively sloped yield curve alone that markets expect $i_{1t} < i^e_{1,t+1}$.

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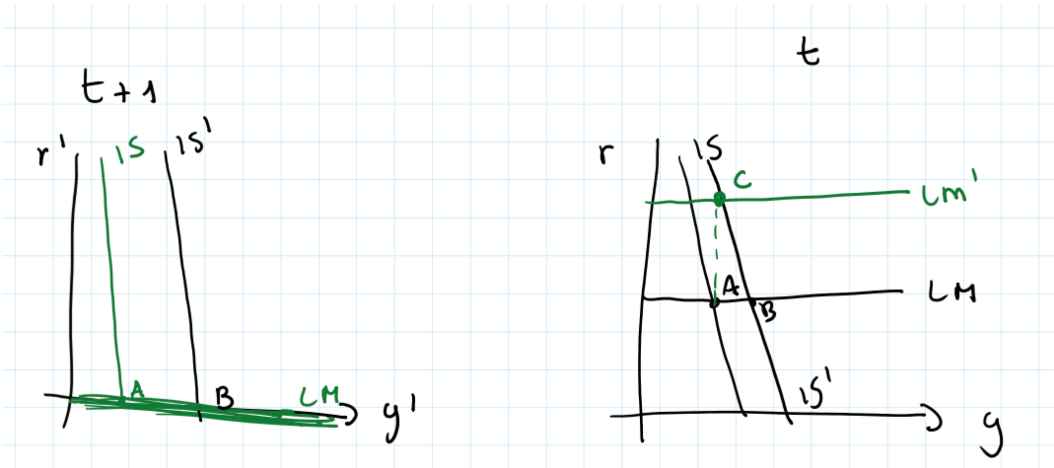
8. Liquidity Trap Case → Expansionary Fiscal Policy at $t+1$, CB Prevents Output Change at t

- introduction and suumptions
    - *Economy in liquidity trap at* $t+1$ *(*$i^e_{1,t+1} = 0$*).*

- *Government announces $G\uparrow$ for t+1.*
- *CB announces it will adjust current rate r to prevent Y from changing at t.*

- **At $t+1$:**
  - $G\uparrow$ → **IS shifts right** → $Y^e_{t+1}\uparrow$. **Interest rate remains at zero** (liquidity trap binds).
- **At t:**
  - $Y^e_{t+1}\uparrow$ → **IS shifts right** → $Y_t\uparrow$.
  - CB intervenes → raises i_{1t} → point C: $Y_t=$ const, $i_{1t}\uparrow$.



Yield curve:

$$i_{2t}=\frac{1}{2}\left(i_{1t}\uparrow+i^e_{1,t+1}=0\right)$$

- Since $i_{1t}>0$ and $i^e_{1,t+1}=0$:
  - $i_{2t}=\frac{1}{2}\,i_{1t}<i_{1t}$ → **negatively sloped yield curve**.
    - slope = $i_{2t}-i_{1t}$ (long term - short term rate)
      - with $i_{2t}=\frac{i_{1t}+0}{2}=\frac{1}{2}i_{1t}$
      - so $i_{2t}<i_{1t}$

Note: even before the announcement, the yield curve was already negatively sloped (because $i_{1t}>0$ and $i^e_{1,t+1}=0$ initially). After the announcement, i_{1t}

rises further, but i_{2t} rises only by half as much. The curve remains negatively sloped but becomes less steep.

- $i^e_{1,t+1} = 0$ means the economy is expected to still be in a **liquidity trap next period**.
- Because the future short rate is zero, the two-period rate is pulled below today's short rate:
    - $i_{2t} = \frac{i_{1t}+0}{2} < i_{1t}$
    - So the inverted yield curve shows that **today's tightening is temporary, while future monetary policy is still constrained by the zero lower bound**.

🗂️ Recap Table

Concept	Key formula / result
Expectations-augmented IS	$Y = A(Y, T, r, Y'^e, T'^e, r'^e) + G$, signs $(+, -, -, +, -, -)$
Why IS is steeper	Change in r alone (with fixed expectations) barely moves PDV of wealth → small ΔA → small ΔY
Standard LM	$r = \bar{r}$ (flat); non-standard: $M/P = Y \cdot L(r)$ (upward sloping)
Two-year yield formula	$i_{2t} = \frac{1}{2}(i_{1t} + i^e_{1,t+1})$
Transitory policy at t	i_{1t} moves; $i^e_{1,t+1}$ unchanged → curve becomes positively sloped (if $i_{1t} downarrow$) or negatively sloped (if $i_{1t} uparrow$)
Policy announced at t, implemented at $t+1$	Both i_{1t} and $i^e_{1,t+1}$ move → slope ambiguous if they move in opposite directions
Positively sloped yield curve	Does NOT necessarily imply $i^e_{1,t+1} > i_{1t}$; risk premium at longer maturities is an alternative explanation
Liquidity trap at $t+1$	$i^e_{1,t+1} = 0$ → yield curve is negatively sloped if $i_{1t} > 0$
Key lesson on policy	Effects of any policy depend on whether and how it changes expectations; if expectations are unchanged, output effects are limited

🌍

Chapter 18 - Openness in goods and financial markets

☑ Reviewed	☐

Context: Extensions – The Open Economy. We now drop the closed-economy assumption and study how openness in goods and financial markets reshapes macroeconomic analysis.

> 💡 **Key message:** In an open economy, agents face two new decisions — whether to buy domestic or foreign goods, and whether to hold domestic or foreign assets. Both choices depend critically on the exchange rate.

---

1. Three Kinds of Openness

Macroeconomics considers three dimensions of openness:

1. **Openness in goods markets**
    - Consumers and firms can choose between domestic and foreign goods.
        - choice affected by:
            a. preferences
            b. price of goods
            c. exchange rates
    - In practice, this freedom is sometimes curtailed by **tariffs** (taxes on imported goods) and **quotas** (restrictions on the quantity of goods that can be imported).
2. **Openness in financial markets**
    - Financial investors can **hold both domestic and foreign assets** (and buy or sell foreign currency on the foreign exchange market).

- Restrictions on such flows are called **capital controls**, which have become much less common among advanced economies in recent decades.

3. **Openness in factor markets**
    - **Firms can choose where to locate production** and **workers can choose where to work**.
    - While **important in the long run**, this dimension plays a much smaller role in the short and medium run. → We therefore **ignore it** throughout.

---

2. Openness in Goods Markets

The Trade Balance

The relationship between a country's exports and imports defines its trade position:

- **Trade deficit**: imports > exports
- **Trade surplus**: imports < exports
- **Trade balance**: imports = exports

Measuring Openness

- The ratio of exports (or imports) to GDP is the most common indicator of openness, but it is an imperfect measure.
- A better index is the share of aggregate output composed of **tradable goods**
    - Goods that compete with foreign goods in either domestic or foreign markets (e.g. cars, computers, pharmaceuticals).
    - **Non-tradable goods** include housing, haircuts, and most local services.
- why do some countries trade more/less?
    a. Typically **a large economy (ex. US) imports and exports less than smaller one**.
    b. **Distance and costs of trading with other economies** (ex. remotes islands in the ocean)
- exports can exceed GDP

- ex. imports=1bn$; value added 0.2bn$; exports=1.2bn$

A country can have a relatively low export-to-GDP ratio and still be highly open if domestic firms are competitive enough to keep imports limited without that appearing as large gross trade volumes.

---

3. The Exchange Rate

When goods markets are open, the key price determining the domestic-vs-foreign choice is the **relative price of domestic goods in terms of foreign goods**: the real exchange rate.

The Nominal Exchange Rate

The **nominal exchange rate** E is the **price of the domestic currency in terms of foreign currency**.

- **Convention used in this course:**
  - taking the **dollar** as the domestic currency and the **pound** as the foreign currency
    - E is the number of pounds one dollar can buy
    - If $E = 0.65$, one dollar buys 0.65 pounds
      - equivalently, $1/E = 1.55$ is the price of a pound in dollars

Changes in E:

- **Nominal appreciation**: $E \uparrow$
  - the domestic currency buys more foreign currency
- **Nominal depreciation**: $E \downarrow$
  - the domestic currency buys less foreign currency

Under a system of **fixed exchange rates**, discrete increases are called **revaluations** and discrete decreases are called **devaluations**.

- mainly discrete changes determined by policy decisions

The Real Exchange Rate

The real exchange rate ε is the **price of domestic goods in terms of foreign goods**. It is constructed as:

$$\varepsilon = \frac{EP}{P^*}$$

where:

- P **is the domestic GDP deflator** (price of domestic goods in domestic currency)
- P^* **is the foreign GDP deflator** (price of foreign goods in foreign currency)
- EP **is therefore the price of domestic goods expressed in foreign currency**.
- Dividing by P^* gives the **relative price of domestic goods in terms of foreign goods.**

Key properties of ε:

- It is an **index number**
  - its level is arbitrary (it depends on the base year chosen for the deflators), but its **rate of change is meaningful**.
    - A 5% increase in ε means domestic goods are 5% more expensive relative to foreign goods than before.
- **Real appreciation** ($\varepsilon \uparrow$)
  - domestic goods become relatively more expensive
    - caused by $E \uparrow$, $P \uparrow$, or $P^* \downarrow$
- **Real depreciation** ($\varepsilon \downarrow$):
  - domestic goods become relatively cheaper
- **Special case**:
  - if $P = P^*$ (same price level in both countries), **then** $\varepsilon = E$**.**
    - ex. Eurozone

In the **short run**, prices are sticky, so P and P^* move slowly. Year-to-year movements in ε are therefore driven largely by movements in E. This is why, **in the short-run model, we can write ε and E as moving together.**

Bilateral vs. Multilateral Exchange Rates

- So far we have considered a single bilateral exchange rate (between two countries).
- In practice, a country trades with many partners, so economists construct the **multilateral real exchange rate**
  - also called the **trade-weighted** or **effective** real exchange rate
  - it is a **weighted average of bilateral real exchange rates**
    - weight of each partner country reflects both **bilateral trade shares** and **competitive pressures in third markets**.

---

4. Openness in Financial Markets

- Openness in financial markets allows investors to hold both domestic and foreign **interest-bearing assets**.
- Holding foreign currency itself is dominated by holding foreign bonds (which also earn interest), so the **relevant decision is between domestic bonds and foreign bonds**.

The Balance of Payments

A country's transactions with the rest of the world are summarised in the **balance of payments**

- the Balance of Payments **is a set of accounts that summarises a country's transactions** with the rest of the world.
  - It has two main parts separated by a line (transactions are described as either *above* or *below* the line).

official lending, government transfers, foreign aid

Net flow of incomes from assets

Export - Import

Current account	
Trade balance (1)	393
Net income (2) (deficit = minus sign)	−76
Current account balance (1) + (2)	317
Capital account (3)	−20
Net lending to/Net borrowing from the rest of the world (1) + (2) − (3)	297
Financial account	
Financial account balance	276
Statistical discrepancy: financial account – current account balance	−21

Flow of assets

1. Current Account (CA) and Capital Account (KA) → above the line

a. **Current Account**

- Records payments to and from the rest of the world:
    - **Trade balance (NX)**: exports minus imports of goods and services
    - **Net income (NI)**: income received on holdings of foreign assets minus income paid to foreigners on their holdings of domestic assets

b. **Capital Account**

- Tracks lending/borrowing from the rest of the world (mainly **foreign aid given/received**), Government transfers,...

- Their sum is the **current account balance**.
    - it yields the total **net lending to / net borrowing from** the rest of the world.

2. Financial Account (FA) → below the line

- Changes in net holdings of foreign assets are recorded in the FA:
    - $CA + KA > 0$: country's net holding of foreign assets increase
        - **exports > imports:** $FA < 0$
            - **foreigners sell foreign asset to pay for their imports.**

- $CA + KA < 0$: country's net holding of foreign assets decrease
  - **exports < imports:** $FA > 0$
    - **"we" sell domestic assets, to pay for our imports.**

Records net capital flows:

> ***Financial account balance***
> $=$
> *foreign holdings of domestic assets* $-$ *domestic holdings of foreign assets*

Also called **net capital flows**:

- **Financial account surplus** (> 0): more capital is flowing in than out
- **Financial account deficit** (< 0): more capital is flowing out than in

Key identity:

- In principle, **a current account deficit must be financed by a financial account surplus.**
  - the country is borrowing from the rest of the world.
- In practice, because current and financial account data come from different sources, a **statistical discrepancy** almost always appears.

GDP vs GNP

In an open economy, two distinct output measures exist:

- **GDP (Gross Domestic Product)**:
  - **value added produced *within the domestic territory***, regardless of who owns the factors
- **GNP (Gross National Product)**:
  - **value added by *domestic factors of production***, regardless of where they are located

The bridge between them is net income from abroad:

$$\text{GNP} = \text{GDP} + \text{NI}$$

(NI = net payments from the rest of the world)

- For most countries the two measures are close.

- Significant divergences arise when large net factor income flows exist
    - for instance:
        - **Kuwait** (large foreign investment income from its sovereign wealth fund)
        - **Ireland** (large profit repatriation by foreign multinationals, making GDP substantially larger than GNP).

---

5. The (Uncovered) Interest Parity Condition

Setup

Consider a US investor choosing between US one-year bonds and UK one-year bonds. The notation:

- i_t: US one-year nominal interest rate
- i_t^*: UK one-year nominal interest rate
- E_t: current nominal exchange rate (dollars per pound, using our convention)
- E_{t+1}^e: expected nominal exchange rate next year

Option 1 — Hold US bonds:

invest $\$1$ today → receive $(1 + i_t)$ dollars next year.

Option 2 — Hold UK bonds:

1. **Convert $\$1$ into pounds** at the current rate: obtain E_t pounds
2. **Invest at the UK rate**: obtain $E_t(1 + i_t^*)$ pounds next year
3. **Convert back to dollars using the expected rate** (each pound is worth $1/E_{t+1}^e$ dollars) and obtain:
    - $E_t(1 + i_t^*)\,\frac{1}{E_{t+1}^e}\ \text{dollars}$

The Arbitrage Condition

- Assuming:

- **investors care only about the expected rate of return**
  - ignoring risk and transaction costs
- **investors will hold only the asset with the highest return**,
- Then:
  - **both assets must yield the same expected return in equilibrium**
    - otherwise everyone would switch to one side.
      - This gives the **uncovered interest parity (UIP) condition**:

$$(1+i_t) = (1+i_t^*)\,\frac{E_t}{E_{t+1}^e}$$

Approximate Form

Rewriting using $\frac{E_t}{E_{t+1}^e} = \frac{1}{1+(E_{t+1}^e - E_t)/E_t}$ and approximating (valid when interest rates and expected depreciation are below ~20% per year):

- considerr:
  - $(1+i_t) = \frac{1+i^*t}{\left(\frac{E^e t+1}{E_t}\right)}$
  - $\frac{E_{t+1}^e}{E_t} = \frac{E_t + E_{t+1}^e - E_t}{E_t} = \frac{E_t}{E_t} + \frac{E_{t+1}^e - E_t}{E_t} = 1 + \frac{E_{t+1}^e - E_t}{E_t}$
- cross terms are small and then negligible (ex. $i * i^*$)

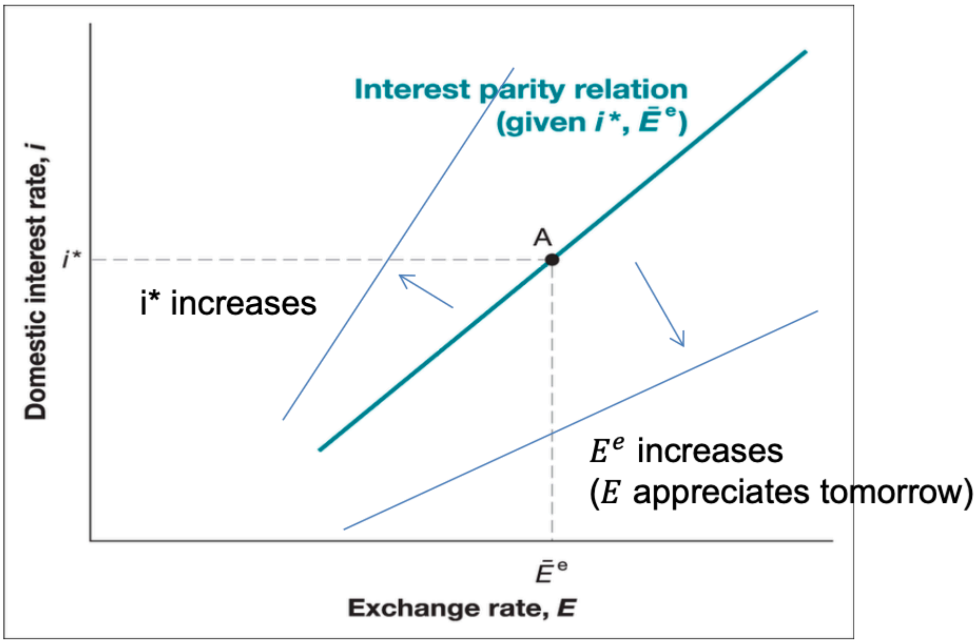
$$i_t \approx i_t^* - \frac{E_{t+1}^e - E_t}{E_t}$$

- Reading this:
  - the **domestic interest rate must equal the foreign interest rate minus the expected appreciation rate of the domestic currency**
    - equivalently, plus the expected depreciation rate of the domestic currency, or minus the expected depreciation rate of the foreign currency

- interest rate parity
  - from $(1+i_t) = (1+i_t^*)\,\frac{E_t}{E_{t+1}^e}$ you can get $E = \frac{1+i}{1+i^*}\bar{E}^e$

- then:

$$i = \left(\frac{1+i^*}{\bar{E}^e}\right) E - 1$$



Intuition (Numerical Example)

Suppose $i_t = 2\%$ (US) and $i_t^* = 5\%$ (UK). Should you buy US or UK bonds?

→ It depends on E_{t+1}^e:

- If you expect the pound to **depreciate by more than 3%** → buy US bonds:
  - the exchange rate loss wipes out the UK interest advantage.
- If you expect the pound to **depreciate by less than 3%** (or even appreciate) → buy UK bonds.

If UIP holds with the US rate 3 percentage points below the UK rate, it must be that investors collectively expect the **dollar to appreciate by about 3%**

- which is precisely why they are willing to hold the lower-yielding US bonds.

Implications for Interest Rate Policy

- Under **fixed exchange rates** ($E_{t+1}^e = E_t$), UIP implies $i_t = i_t^*$:
  - **Domestic and foreign interest rates must move exactly together**.

- More generally, if a **country wants to avoid large exchange rate movements**, it severely **limits its freedom to set domestic interest rates independently of rates abroad.**

🗂️ Recap Table

Concept	Key formula / definition	What it says
Nominal exchange rate	E: price of domestic currency in foreign currency	Appreciation = $E\uparrow$; depreciation = $E\downarrow$
Real exchange rate	$\varepsilon = EP/P^*$	Relative price of domestic goods in terms of foreign goods; moves with E in the short run
Real appreciation / depreciation	$\varepsilon\uparrow$ / $\varepsilon\downarrow$	Domestic goods become relatively more / less expensive
Current account balance	Trade balance (NX) + Net income (NI)	Net payments received from the rest of the world
Financial account balance	Foreign holdings of domestic assets $-$ domestic holdings of foreign assets	CA deficit must be financed by financial account surplus (net capital inflow)
GNP	$\mathrm{GNP} = \mathrm{GDP} + \mathrm{NI}$	GDP adjusted for net factor income flows from abroad
UIP condition (exact)	$(1+i_t) = (1+i_t^*)\,E_t/E_{t+1}^e$	Arbitrage equalises expected returns on domestic and foreign bonds
UIP condition (approx.)	$i_t \approx i_t^* - (E_{t+1}^e - E_t)/E_t$	Domestic rate = foreign rate minus expected domestic appreciation

🌐

Chapter 19 - The goods market in an open economy

☑ Reviewed ☐

Reference: Lectures 23, 24, 25 and Chapter 19 Textbook (pp. 404–420)

Context: Extensions – The Open Economy. We extend the goods-market model of Chapter 3 to account for openness: part of domestic demand falls on foreign goods, and part of the demand for domestic goods comes from abroad.

> 💡 **Key message:** In an open economy, output depends on both domestic and foreign demand. Shocks abroad affect us through exports, and our policy choices affect others through imports.

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1. 📦 Z — Demand for Domestic Goods (The IS Relation in the Open Economy)

In a **closed economy**, domestic demand and demand for domestic goods are the same thing. In an **open economy** they **diverge**, and we must track both separately.

We define three curves:

- **DD — Domestic Demand for all goods:**

  $$DD = C(Y - T) + I(Y, r) + G$$

  - Total spending by domestic residents on both domestic and foreign goods.
  - If the economy were closed, DD would equal ZZ.
- **AA — Domestic Demand for Domestic Goods:**

$$AA = C(Y - T) + I(Y, r) + G - \frac{IM}{\varepsilon}(Y, \varepsilon)$$

  - We **subtract the part of domestic demand that falls on foreign goods**.
- **ZZ — Total Demand for Domestic Goods:**

$$ZZ = AA + X(Y^*, \varepsilon)$$

  - We **add exports**, i.e. the foreign demand for domestic goods.

The **goods-market equilibrium** ($Y = Z$) in the open economy is therefore the **IS relation**:

$$Y = C(Y - T) + I(Y, r) + G - \frac{IM}{\varepsilon}(Y, \varepsilon) + X(Y^*, \varepsilon)$$

- Defining **net exports** $NX = X - \dfrac{IM}{\varepsilon}$, this can be written compactly as:
  - $Y = C + I + G + NX.$

Imports

$IM = IM(Y, \varepsilon)$ with signs $(+, +)$:

1. **Domestic income** Y $(+)$:
   - **higher income raises demand for all goods**, domestic and foreign → $IM \uparrow$
2. **Real exchange rate** ε $(+)$:
   - **higher** ε **means domestic goods are more expensive relative to foreign goods**, i.e. foreign goods are cheaper → $IM \uparrow$

- $\dfrac{IM}{\varepsilon}$ is the **value of imports in terms of domestic goods**
  - because $\dfrac{1}{\varepsilon}$ is the price of foreign goods expressed in domestic goods.

Note:

- when $\varepsilon \uparrow$, both $IM \uparrow$ and $\dfrac{1}{\varepsilon} \downarrow$, so the net effect on $\dfrac{IM}{\varepsilon}$ is ambiguous

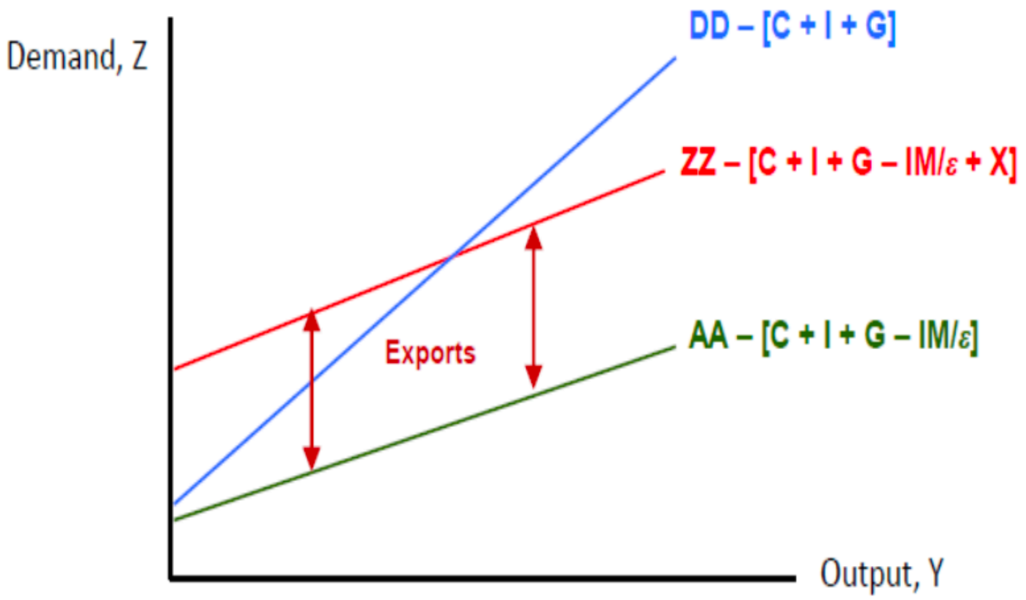
- resolved later via the Marshall-Lerner condition.

Exports

$X = X(Y^*, \varepsilon)$ with signs $(+, -)$:

1. **Foreign income Y^*** $(+)$:
   - higher foreign income raises foreign demand for domestic goods → $X \uparrow$.
2. **Real exchange rate ε** $(-)$:
   - higher ε makes domestic goods more expensive for foreigners → $X \downarrow$.

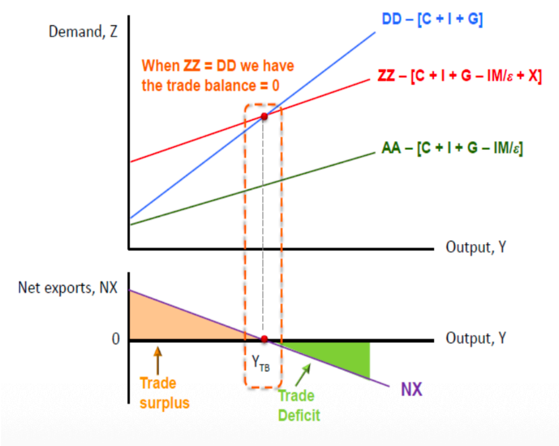
Graphical Analysis

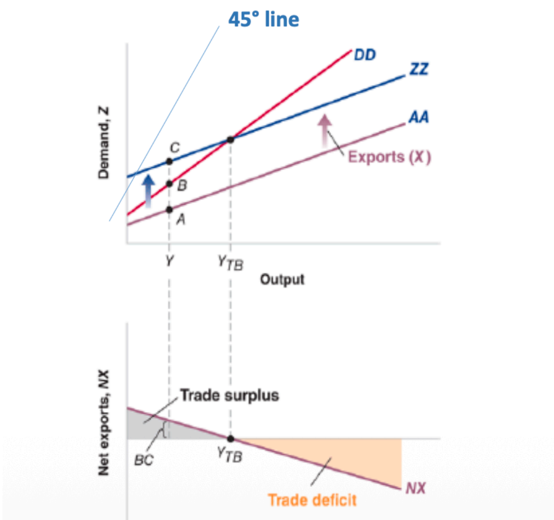


- **DD** has a positive slope < 1:
  - Income rises → demand rises but less than one-for-one.
- **AA** is **flatter than DD**:
  - **As $Y \uparrow$, some additional domestic demand spills onto imports rather than domestic goods.**

- The vertical gap between DD and AA equals $\frac{IM}{\varepsilon}$, <u>which **widens** with Y</u>:

- **ZZ is parallel to AA and above it**:
  - The <u>**gap equals exports** </u>X, which does **not** depend on domestic income Y
    - only on Y^* and ε
  - Since AA is flatter than DD, ZZ is also flatter than DD.





Identifying components on the graph:

- at any output level Y, exports $= AC$, imports $= AB$, net exports $= BC$.

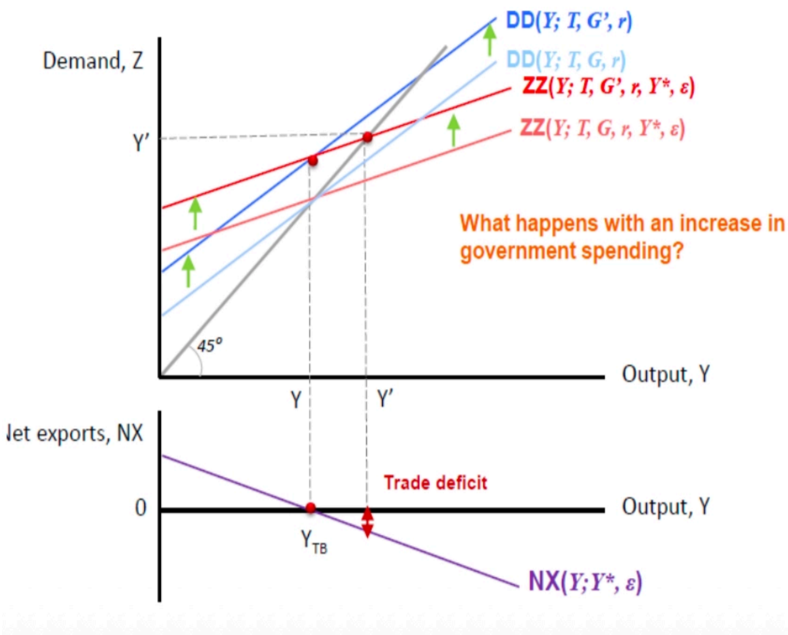
<u>Trade balance and the NX line</u>:

- Y_{TB} (trade-balance output) is found at the **intersection of DD and ZZ**
  - the output level at which $NX = 0$.
- The **NX line** is **downward sloping**:
  - as $Y \uparrow$, **imports rise and exports are unaffected** → $NX \downarrow$.
- $Y > Y_{TB}$ → **trade deficit**; $Y < Y_{TB}$ → **trade surplus**.

> **Policy checklist:** when analysing any open-economy shock, always track the effect on: DD, ZZ, NX line, Y_{TB}, equilibrium Y, and final NX.

Intersection	Economic Meaning	Key Equation
ZZ and 45° line	Market Equilibrium	Y = Z
ZZ and DD line	Balanced Trade (Y_{TB})	NX = 0 (or $X = IM/\epsilon$)

Fiscal Policy — Increase in Government Spending ($G\uparrow$)



Starting from trade balance ($Y = Y_{TB}$), suppose $G\uparrow$ by ΔG:

- **ZZ shifts up** by ΔG → new equilibrium Y' is higher.
- **DD shifts up** by ΔG by the same amount.
- **NX line does not shift** (NX depends on Y, Y^*, ε — not on G) → Y_{TB} is unchanged.
    - The movement is a slide along the curve, not a shift.

- Since both DD and ZZ shift up by the same amount, their intersection (which gives Y_{TB}) remains at the same output level.
- **Result:** $Y\uparrow$, but since $Y' > Y_{TB}$, we end up in a **trade deficit**.

The open-economy multiplier is smaller than in a closed economy.

- **Part of the income generated leaks out into imports.**
- Letting m be the marginal propensity to import:

> $$\frac{\Delta Y}{\Delta G} = \frac{1}{1 - c_1 + \frac{m}{\varepsilon}}$$

  - This is **smaller than the closed-economy multiplier** $\frac{1}{1 - c_1}$.
    - The **more open the economy** (higher m), the **smaller the multiplier** and the **larger the trade deficit that results from a fiscal expansion**.
- → SO:
  - **in an open economy an increase in domestic demand has a smaller effect on output than in a closed economy and has a negative effect on the trade balance** (the trade balance worsens/deteriorates).

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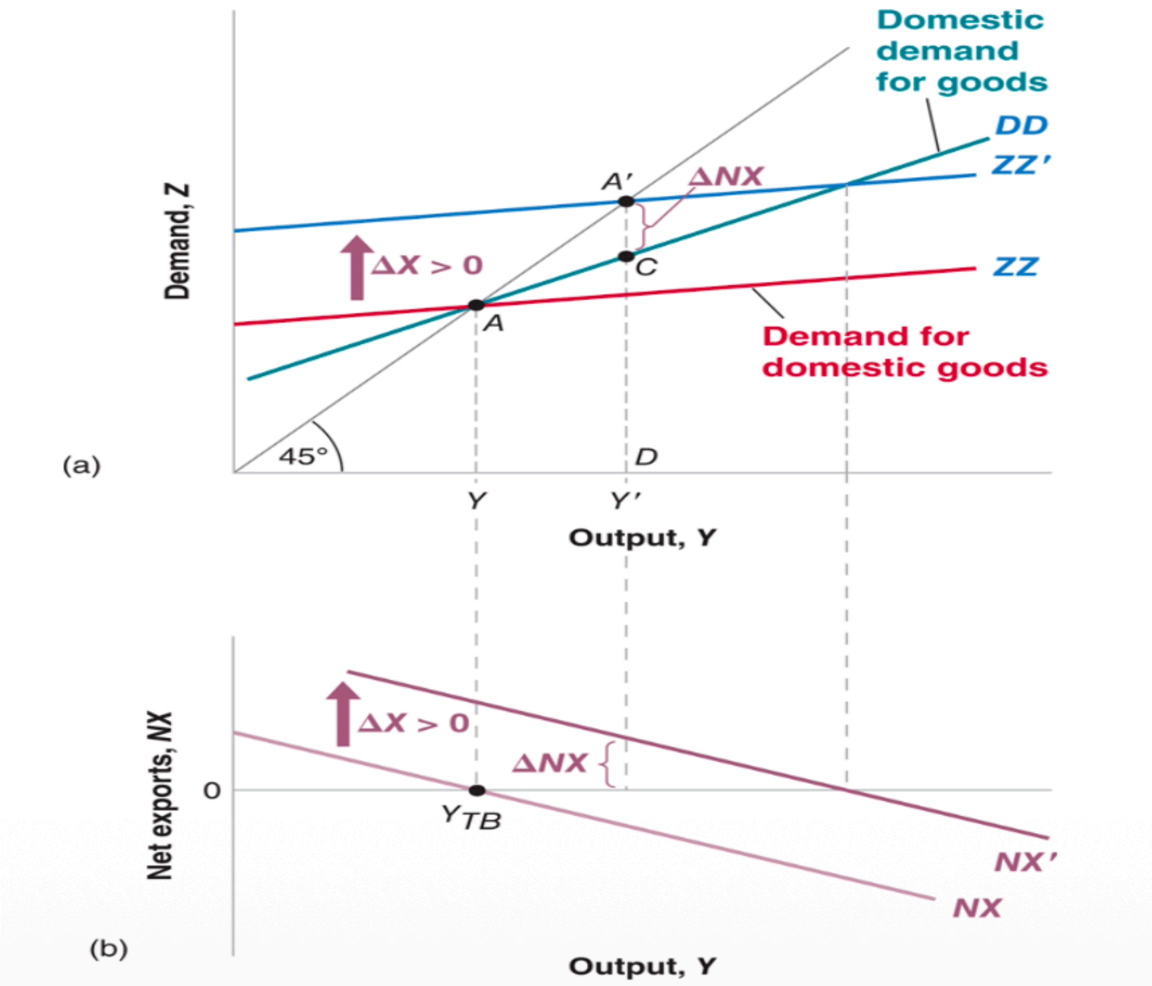
2. 🌐 Increase in Foreign Output / Demand



Suppose foreign output $Y^* \uparrow$ (for any reason, e.g. a fiscal expansion abroad):

- **Exports increase** by ΔX → **ZZ shifts up** by ΔX.
- **DD does not move**:
  - Y^* does not directly affect domestic C, I or G.
- **NX line shifts up** by ΔX:
  - for any given domestic Y, net exports are now higher → $Y_{TB} \uparrow$.
    - $NX = X(Y^*, \epsilon) - \frac{IM(Y,\epsilon)}{\epsilon}$
      - where $X(Y^*, \epsilon)$ defines the intercept
- **Result:** $Y \uparrow$ and the **trade balance improves** (trade surplus if starting from balance).

Why does the trade balance improve?

- **Even though higher Y raises imports, the initial export boost dominates.**
    - At the new equilibrium Y', domestic demand (DC in the image) lies below demand for domestic goods (DA'), so net exports $CA' > 0$.

Key contrast with a domestic demand shock:

Shock	Y	NX	Trade balance
Domestic demand ↑ ($G \uparrow$)	↑	↓	deficit
Foreign demand ↑ ($Y^* \uparrow$)	↑	↑	surplus

Policy Coordination and the Free-Rider Problem

Since demand shocks in one country spill over to others via trade, countries in recession should in principle **coordinate fiscal expansions simultaneously**:

- If all expand together, each benefits from **higher exports** as well as **domestic demand**, so the **trade balance does not deteriorate individually**.
- However, each country has an incentive to **free-ride**:
    - wait for others to expand, benefit from the rise in their own exports, and avoid the cost of fiscal expansion and the associated trade deficit.
- If every country waits, nothing happens and the recession persists.

---

3. 🔄 Real Exchange Rate and Depreciation

The real exchange rate is defined as:

$$\varepsilon = \frac{EP}{P^*}$$

In the **short run**, prices P and P^* are taken as given, so a **nominal depreciation ($E \downarrow$) translates one-for-one into a real depreciation ($\varepsilon \downarrow$)**:

- domestic goods become cheaper relative to foreign goods.

A real depreciation affects $NX = X(Y^*, \varepsilon) - \frac{IM}{\varepsilon}(Y, \varepsilon)$ through **three channels**:

1. **Export effect** [$\varepsilon \downarrow \Rightarrow X \uparrow \Rightarrow NX \uparrow$]:
    - **domestic goods are now cheaper for foreigners**

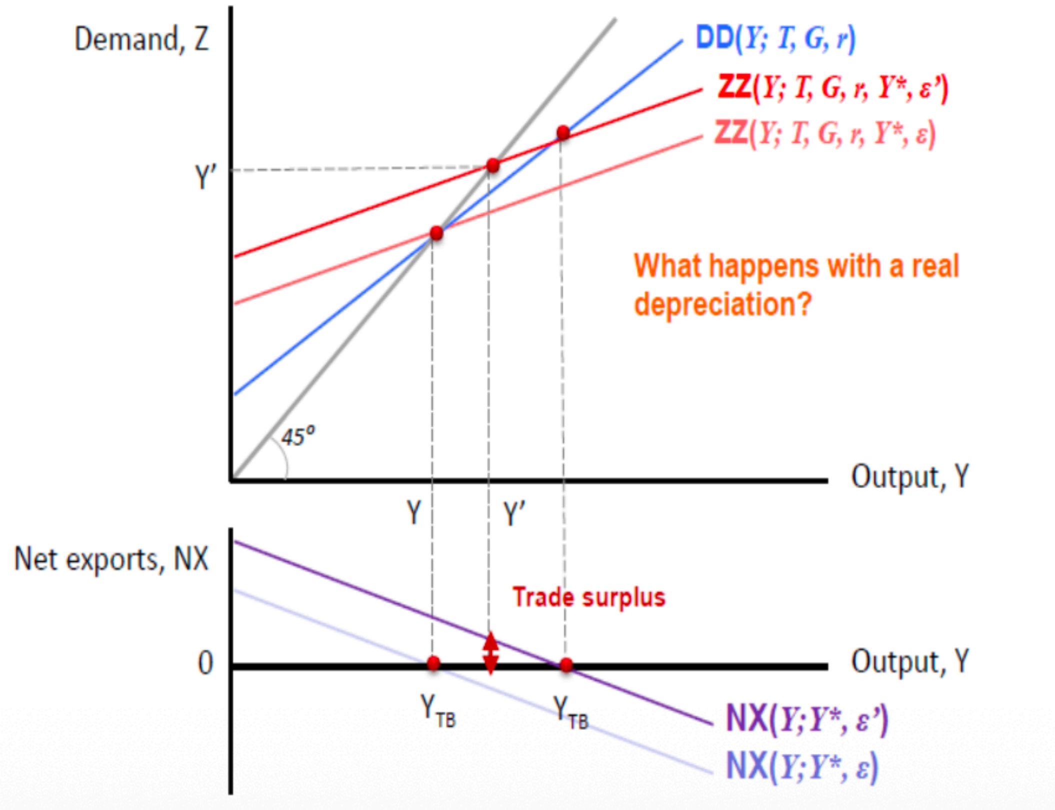
- → foreign demand for domestic goods rises.

2. **Import quantity effect** [$\varepsilon \downarrow \Rightarrow IM \downarrow \Rightarrow NX \uparrow$]:
   - **foreign goods are now relatively more expensive at home**
     - → domestic residents shift toward domestic goods.
3. **Price effect** [$\varepsilon \downarrow \Rightarrow \frac{1}{\varepsilon} \uparrow \Rightarrow NX \downarrow$]:
   - the **relative price of foreign goods in terms of domestic goods rises**.
     - The same quantity of imports now costs more to buy in domestic goods
       - this is the **adverse price effect** of depreciation.

Effects 1 and 2 push **NX up**; effect 3 pushes **NX down**.

- The Marshall-Lerner condition (Section 5) determines which dominates.

Graphical Effects of Real Depreciation (assuming M-L holds)



Starting from trade balance, suppose $\varepsilon \downarrow$:

- **ZZ shifts up** (exports rise, M-L assumed).
- **DD does not move**.
- **NX line shifts up** → $Y_{TB} \uparrow$.
- **Result:** $Y \uparrow$, **trade balance improves**.

The graphical effect mirrors that of an increase in Y^*, **BUT** there is an important **distributive difference**:

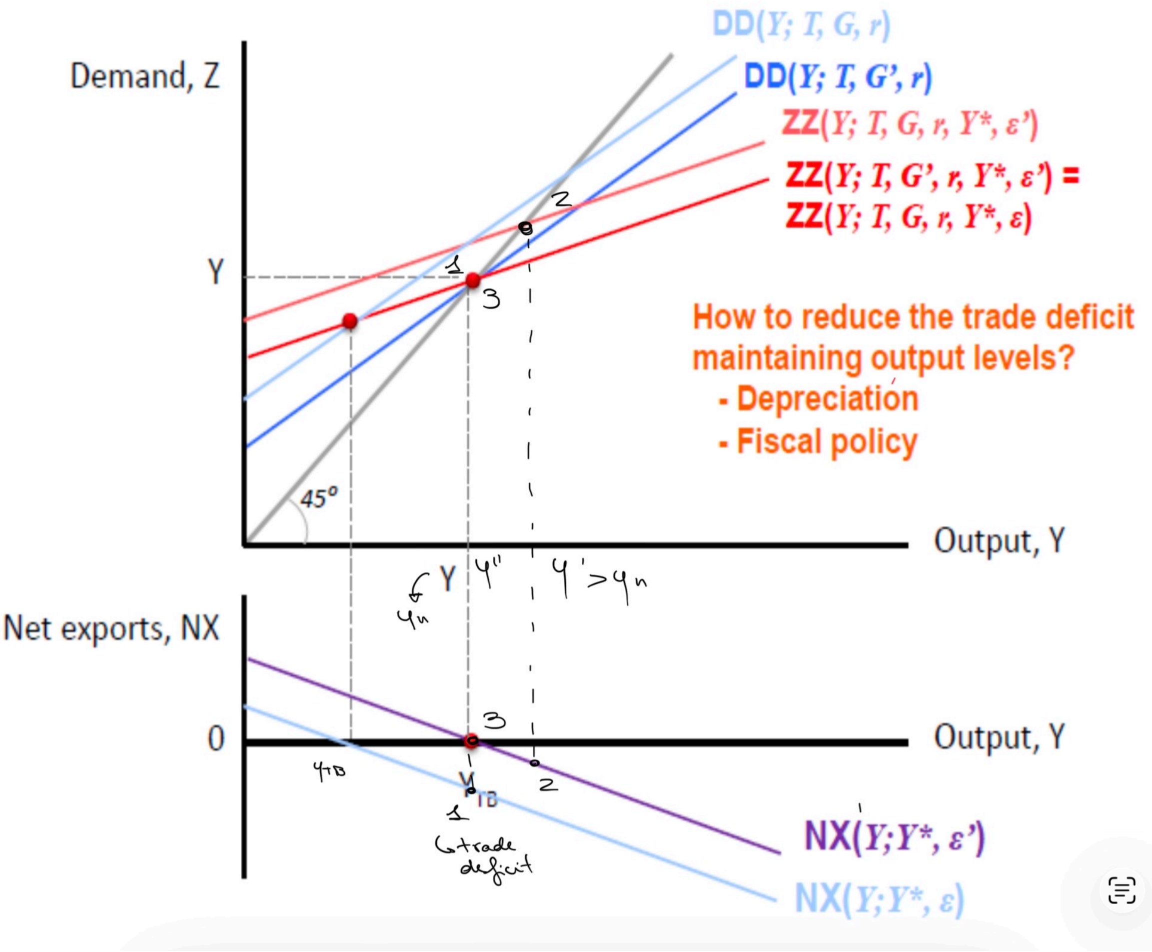
- depreciation makes **imports more expensive**, so **citizens who rely heavily on foreign goods are worse off in real terms even as aggregate output rises**.
- This mechanism can generate **social unrest**
  - as seen in Mexico after the 1994–5 peso depreciation

---

4. 🎯 Policy Mix — Combining Fiscal and Exchange Rate Policies

A government with two targets (output level and trade balance) requires **two instruments** (fiscal policy and the exchange rate). The general principle: **two targets, two instruments**.

Example: large trade deficit at $Y = Y_n$



Goal: ***reduce the trade deficit** while **keeping output unchanged***.

NX depends on Y (which must stay constant), Y^* (uncontrollable), and ε.

→ Only ε is available to improve NX without touching Y^*.

- **Step 1 — Real depreciation ($\varepsilon \downarrow$):**
  - via the M-L condition, $NX \uparrow$ and exports rise
    - → but this also raises Y above Y_n, which is undesired.
- **Step 2 — Fiscal contraction ($G \downarrow$ and/or $T \uparrow$):**
  - shifts ZZ′ back down to ZZ to offset the output increase, bringing Y back to Y_n.
- **Result:** same output, improved trade balance.

Summary table (exchange rate and fiscal policy combinations):

Initial conditions	Trade surplus	Trade deficit
Low output	ε ?, $G \uparrow$	$\varepsilon \downarrow$, G ?
High output	$\varepsilon \uparrow$, G ?	ε ?, $G \downarrow$

Where "?" indicates the direction depends on the specific magnitudes in the economy.

Example: output below natural level and trade deficit

A real depreciation can serve both purposes simultaneously:

- it raises Y toward Y_n and improves NX via the M-L condition.
  - No fiscal policy mix is needed if the depreciation alone achieves both targets.

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5. ⚖️ Marshall-Lerner (M-L) Condition

The Marshall-Lerner condition answers the question: does a real depreciation improve net exports?

> **Marshall-Lerner condition:**
> **A real depreciation ($\varepsilon \downarrow$) leads to $NX \uparrow$** if the increase in exports and the decrease in imports are large enough to more than offset the adverse price effect.
>
> → i.e. effects 1 and 2 (export and import quantity effects) dominate effect 3 (price effect.

> **Marshall-Lerner condition:**
> a **real depreciation ($\varepsilon \downarrow$) leads to $NX \uparrow$** if and only if the **sum of export and import price elasticities exceeds one.**
> → i.e. if the increases in X and decreases in IM are large enough to more than offset the adverse price effect on the import bill.

Empirical evidence: the condition holds in practice — but the timing matters:

- **Short run:**
  - M-L tends **not** to hold.
  - Quantities of X and IM are slow to react (firms have existing contracts, consumers take time to switch suppliers).

- Only the price effect operates immediately → NX initially **worsens**.
- **Medium run:**
  - M-L holds.
  - Over time, $X\uparrow$ and $IM\downarrow$ sufficiently to dominate the price effect → $NX\uparrow$.

When M-L fails — Exercise analysis

Exercise 13 (solved in class): Consider an economy where **imports do not depend on** ε, i.e. $IM = IM(Y)$. Then:

$$NX(Y^*, Y, \varepsilon) = X(Y^*, \varepsilon) - \frac{IM(Y)}{\varepsilon}$$

When $\varepsilon\downarrow$:

- $X\uparrow$ → $NX\uparrow$ (favourable)
- But $\frac{1}{\varepsilon}\uparrow$ → $\frac{IM}{\varepsilon}\uparrow$ → $NX\downarrow$ (price effect)

The net effect is **uncertain** — M-L is **not** guaranteed.
The statement "M-L always holds here" is **false**.

Exercise 14 (homework): In an economy where NX does not depend on ε at all (neither X nor IM respond), a depreciation only triggers the price effect. X stays the same, import quantities stay the same, but $\frac{IM}{\varepsilon}\uparrow$ → NX deteriorates. M-L does **not** hold.

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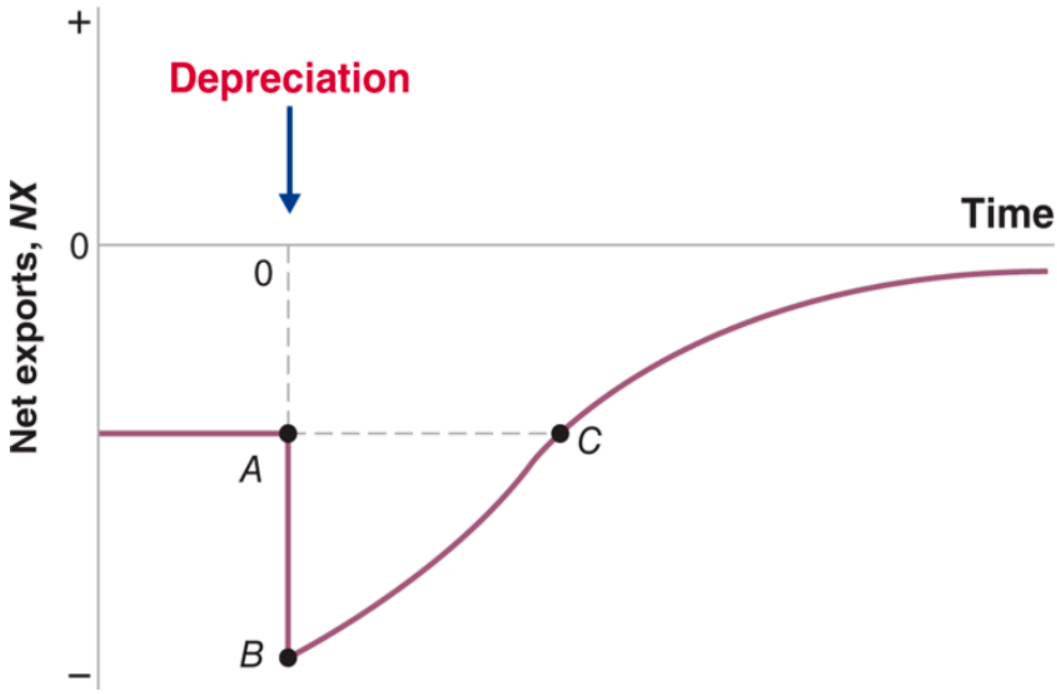
6. 〰️ The J-Curve



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The <u>**J-curve**</u> describes the **dynamic path of the trade balance following a real depreciation over time**:

- **Immediately (short run — point A → B):**
  - 0A: pre-depreciation trade deficit
    - at first the deficit <u>**worsen**</u> (0B)
  - Prices react faster than quantities.
  - Import quantities are slow to adjust (existing contracts, search costs, habits).
  - The depreciation immediately raises $\frac{1}{\varepsilon}$ → each unit of imports costs more in domestic goods → ‘NX <u>**worsens**</u>. The <u>M-L condition does not hold in the short run.</u>
- **Over time (medium run — point B → C and beyond):**
  - Firms and consumers gradually adjust.
  - **Exports increase** as foreign buyers respond to lower prices; **import quantities fall** as domestic buyers substitute away from foreign goods.

- If M-L holds, NX eventually **improves beyond its initial pre-depreciation level**.

The resulting path traces out a **J shape**:

- An initial dip followed by a sustained improvement.
- Empirical data confirm lags of roughly **6 months to 1 year** before the improvement materialises.

> **Summary:** depreciation initially reduces NX (price effect dominates in the short run); over time, the quantity effects dominate and NX improves. In the case of an appreciation, the opposite J-curve logic applies.

---

7. 💰 Saving, Investment and the Current Account Balance

The **goods-market equilibrium** can be rewritten in terms of **saving and investment**, an alternative perspective that reveals the link between fiscal policy, trade and external lending.

Derivation (without net income)

Start from: $Y = C + I + G + NX$

Subtract C and T from both sides:

$$Y - C - T = I + (G - T) + NX$$

The left side is **private saving** $S = Y - C - T$. Using $S_{pub} = T - G$:

$$S = I - S_{pub} + NX \quad \Longrightarrow \quad S + S_{pub} = I + NX$$

This is the **goods-market equilibrium rewritten as national saving equals investment plus net exports.** Denoting $S_{NAT} = S + S_{pub}$:

$$S_{NAT} = I + NX$$

Extending to the Current Account (with net income from abroad)

In an **open economy,** the total income of domestic residents is $Y + NI$ (domestic output plus net income from abroad). Adding NI to both sides and subtracting taxes:

$$(Y + NI - T) - C = I + (G - T) + (NX + NI)$$

- $(Y + NI - T) - C = S$
- $(Y + NI - T) - C - I - (G - T) = (NX + NI)$

The left side is again private saving S. Defining the **Current Account** $CA = NX + NI$:

> $CA = S + (T - G) - I = S_{NAT} - I$

Interpretation

- $S_{NAT} > I$ → **CA surplus**:
  - the country saves more than it invests → it is a **net lender** to the rest of the world.
- $S_{NAT} < I$ → **CA deficit**:
  - the country saves less than it invests → it is a **net borrower** from the rest of the world.

Key implications of the identity:

- An **increase in investment** must be matched by an increase in private or public saving, or a **deterioration of the CA**.
- A **deterioration in the government budget** ($G \uparrow$ or $T \downarrow$, i.e. $T - G \downarrow$) must be offset by higher private saving, lower investment, or a **CA deterioration**
  - this last case is the **"twin deficits"** phenomenon.
- A **high-saving country** must have either a high investment rate or a large CA surplus.

Important caveat: the identity does not tell us *which* component adjusts — that requires the full model. For example, a depreciation improves NX and hence CA by boosting output and saving, even though the identity alone does not make this mechanism visible.

🗂️ Recap Table

Concept	Key formula / result	Direction / remark
IS in open economy	$Y = C(Y - T) + I(Y, r) + G - \frac{IM}{\varepsilon} + X$	Equilibrium condition

Concept	Key formula / result	Direction / remark
Open-economy fiscal multiplier	$\frac{1}{1-c_1+m/\varepsilon}$	Smaller than closed-economy $\frac{1}{1-c_1}$
Domestic demand ↑ ($G\uparrow$)	$Y\uparrow$, $NX\downarrow$	Trade deficit; Y_{TB} unchanged
Foreign demand ↑ ($Y^*\uparrow$)	$Y\uparrow$, $NX\uparrow$	Trade surplus; $Y_{TB}\uparrow$
Real depreciation ($\varepsilon\downarrow$) if M-L holds	$Y\uparrow$, $NX\uparrow$	Trade surplus; similar graph to $Y^*\uparrow$
Real exchange rate	$\varepsilon = EP/P^*$	In short run: $E\downarrow\Leftrightarrow\varepsilon\downarrow$
Marshall-Lerner condition	$\varepsilon\downarrow\Rightarrow NX\uparrow$ if export + import elasticities > 1	Holds in medium run; not in short run
J-curve	NX worsens initially, then improves	Lag of ~6–12 months empirically
National saving = I + NX	$S+S_{pub}=I+NX$	Goods-market equilibrium, rewritten
Current account	$CA=S_{NAT}-I$	Surplus → net lender; deficit → net borrower
Twin deficits	$T-G\downarrow$ → $CA\downarrow$ (if S, I unchanged)	Budget deficit + CA deficit

💱

Chapter 20 - Output, the interest rate and the exchange rate

☑ Reviewed	☐

Reference: Lectures 25, 26 and 27 (8 and 10 April 2025) and Chapter 20 Textbook (pp. 425–443, appendix included)

Context: Extensions – The Open Economy. We now extend the IS-LM framework to an open economy by adding a third market: the foreign exchange market. The exchange rate is no longer a policy instrument but an endogenous variable determined by equilibrium in financial markets.

> 💡 **Key message:** The Mundell-Fleming model (IS-LM-UIP) shows that the effects of monetary and fiscal policy differ sharply depending on the exchange rate regime. Under fixed exchange rates, monetary policy becomes inoperative; under flexible rates, it gains additional power through the exchange rate channel.

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1. 📦 The Goods Market — IS Relation in the Open Economy

In Chapter 19 we treated the exchange rate as exogenous. Now we endogenise it. We start by recalling the open-economy goods-market equilibrium:

$$Y = C(Y - T) + I(Y, r) + G - \frac{IM}{\varepsilon}(Y, \varepsilon) + X(Y^*, \varepsilon)$$

To simplify and focus on the short run, we make the following **assumptions**:

- $P = P^*$ are taken as given, and we normalise $P/P^* = 1$, so the **real and nominal exchange rates coincide**: $\varepsilon = \dfrac{EP}{P^*} = E$.

- No expected inflation: $\pi^e = 0$, so $r = i$ (**nominal = real interest rate**).

Under these simplifications the IS relation for the open economy becomes:

$$Y = C(Y - T) + I(Y, i) + G + NX(Y, Y^*, E)$$

This is the **new open-economy IS relation**

- identical in structure to the closed-economy IS, but with net exports NX now depending on:
  - domestic output Y (negatively, higher Y raises imports)
  - foreign output Y^* (positively, higher Y^* raises exports)
  - nominal exchange rate E (negatively, appreciation reduces NX, assuming the Marshall-Lerner condition holds).

---

2. 💹 Uncovered Interest Parity (UIP)

The Condition

- In an open financial market, investors choose between domestic and foreign bonds.
- **Arbitrage** ensures that, in equilibrium, **both must yield the same expected return expressed in the same currency**. This is the **Uncovered Interest Parity (UIP) condition**:

$$(1 + i_t) = (1 + i_t^*)\,\frac{E_t}{E_{t+1}^e}$$

- The left-hand side is the gross return from holding a domestic bond.
- The right-hand side is the expected gross return, in domestic currency, from holding a foreign bond:
  - you earn $(1 + i^*)$ in foreign currency, but then convert back at the expected future rate, which introduces the ratio E_t / E_{t+1}^e.

Rearranging to express the **current exchange rate**:

$$E_t = \frac{1 + i_t}{1 + i_t^*}\,E_{t+1}^e$$

Taking the expected future exchange rate $\bar{E}^e$ as given and dropping time subscripts, the UIP simplifies to:

$$E = \frac{1+i}{1+i^*}\,\bar{E}^e$$

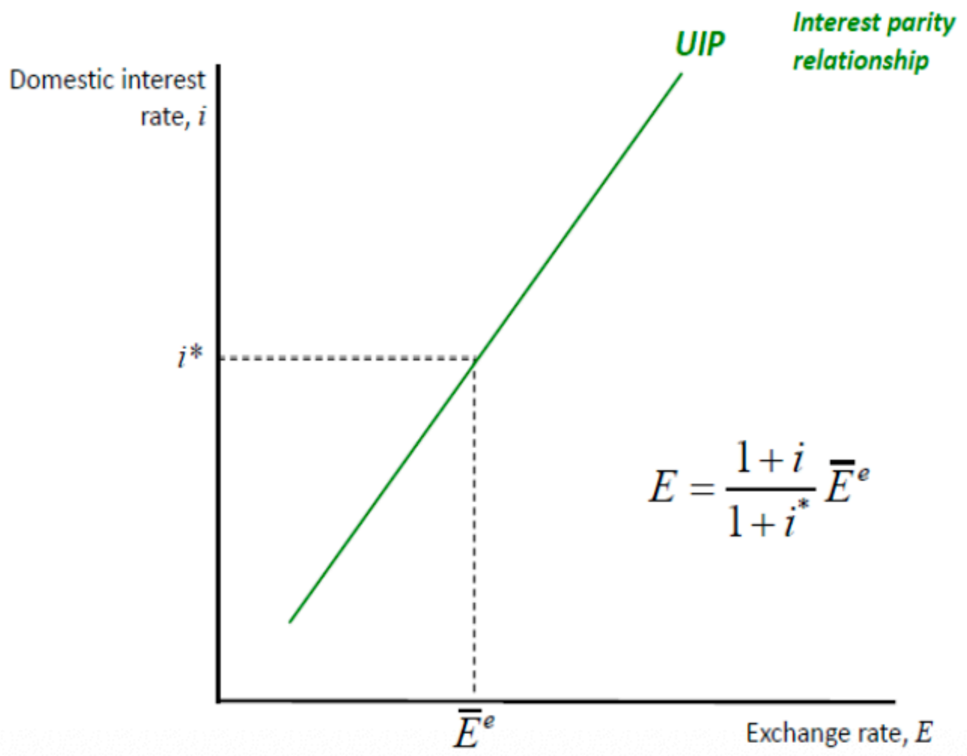
The current exchange rate therefore depends on three things:

1. the **domestic interest rate** i
2. the **foreign interest rate** i^*
3. the **expected future exchange rate** $\bar{E}^e$

What drives E? → Intuition

- $i \uparrow$:
  - domestic assets become more attractive → capital inflows → demand for domestic currency ↑ → **appreciation** ($E \uparrow$).
- $i^* \uparrow$:
  - foreign assets become more attractive → capital outflows → demand for foreign currency ↑ → **depreciation** ($E \downarrow$).
- $\bar{E}^e \uparrow$:
  - investors expect the domestic currency to be worth more in the future → they buy domestic assets now → $E \uparrow$ today (**appreciation**).

Graph of the UIP Relation (i vs E)



Rewriting UIP with i on the vertical axis:

$$i = \frac{E(1+i^*) - \bar{E}^e}{\bar{E}^e}$$

This gives a **positively sloped line** in (E, i) space:

- **Slope:** $\frac{1+i^*}{\bar{E}^e} > 0$
- **Vertical intercept** (set $E = 0$): $i = -1$
  - the UIP cannot shift, only rotate
- **Horizontal intercept** (set $i = 0$): $E = \frac{\bar{E}^e}{1+i^*}$

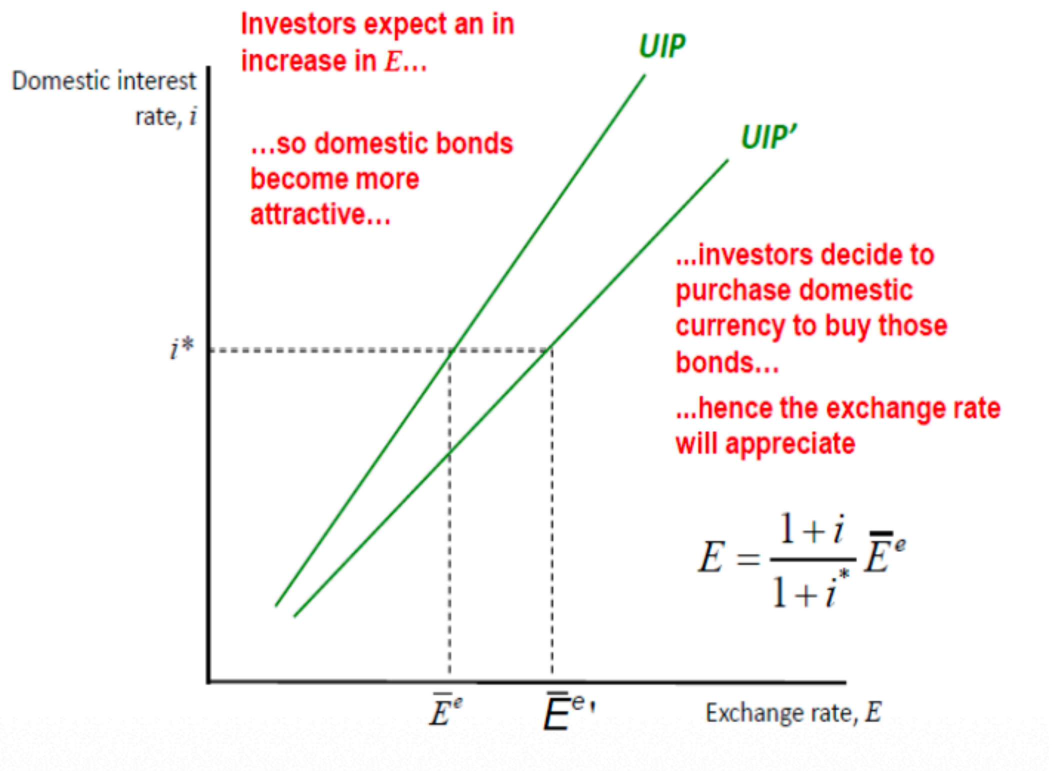
What shifts vs. what rotates the UIP?

- A change in the **domestic interest rate** i:
  - movement *along* the UIP curve

- A change in i^* (foreign rate):
  - **rotates** the UIP (changes the slope)

    - when $i^* \downarrow$, the slope becomes flatter and the horizontal intercept falls
        - at any given i, investors now demand a higher E to remain indifferent

- A change in $\bar{E}^e$ (suppose $\bar{E}^e$ increases):
    - **rotates** the UIP to the right (both slope and intercept change)
    - an expected future appreciation means investors value the domestic currency more today, appreciating it



- The UIP line **does not shift** in a parallel manner, it always pivots through the vertical intercept at $i = -1$.

> ⚠️ A common mistake: when i^* or $\bar{E}^e$ change, the UIP **rotates**, it does not shift. Only if the vertical intercept itself changed would we call it a shift.

---

3. 💰 LM — Standard Version

In the standard version of the model, the central bank sets the policy interest rate directly:

> $i = \bar{i}$

The LM is therefore a **horizontal line** in (Y, i) space, exactly as in the closed-economy standard IS-LM model. The central bank's instrument is $\bar{i}$; moving $\bar{i}$ shifts the LM up or down.

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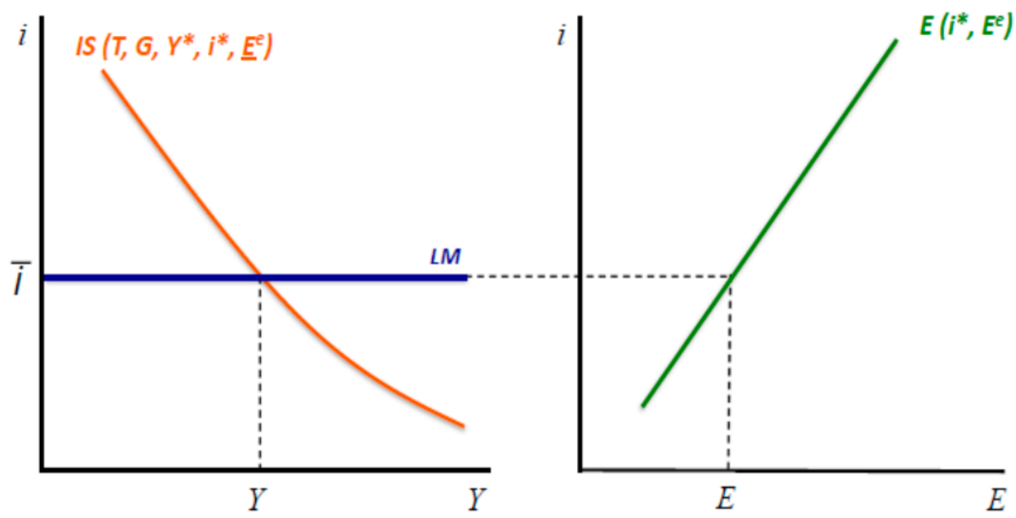
4. 🔗 IS-LM-UIP Together — The Mundell-Fleming Model (Standard Version)



The System

Combining the three markets — goods (IS), financial (LM), and foreign exchange (UIP), we obtain the **standard IS-LM-UIP model**, also known as the **Mundell-Fleming model**:

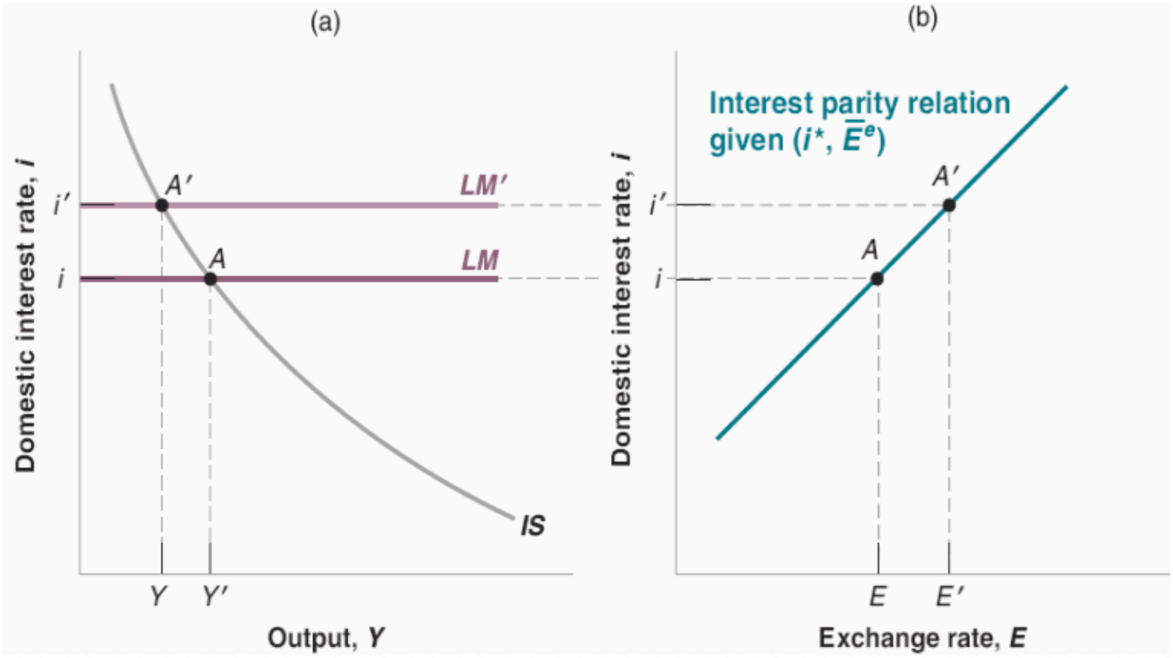
$$IS: \quad Y = C(Y-T) + I(Y,i) + G + NX\left(Y, Y^*, \frac{1+i}{1+i^*}\bar{E}^e\right) LM: \quad i = \bar{i}$$

The UIP is used to substitute E out of the IS:

- **once we know i from the LM, the UIP immediately pins down E.**

- So the model has two equations and two unknowns (Y and i), with E determined residually

Why Is the IS Downward Sloping?



An increase in the interest rate i reduces output Y through **two channels**:

1. **Investment channel** (same as closed economy): $i \uparrow \Rightarrow I \downarrow \Rightarrow Z \downarrow \Rightarrow Y \downarrow$.
2. **Exchange rate channel** (open economy only): $i \uparrow \Rightarrow E \uparrow$ (appreciation) $\Rightarrow X \downarrow, IM \uparrow \Rightarrow NX \downarrow \Rightarrow Y \downarrow$ (via Marshall-Lerner).
   - for this second channel to work, we **assume the Marshall-Lerner (ML) condition is satisfied** → this ensures that the appreciation reduces NX rather than increasing it.

   Both channels push in the same direction: **the IS is downward sloping**.

   > 💡 **Special case:** even if I is fully exogenous ($I = \bar{I}$), the IS is **still downward sloping** because of channel 2. The IS becomes **vertical only** when *both* I is fully exogenous *and* NX does not depend on ε (i.e., the UIP channel is also severed).

- **ambiguity of NX**

- While the IS curve definitely slopes down, the total effect on NX at the new equilibrium is mathematically uncertain because of two competing forces:
  - **The Appreciation Effect:**
    - Because ML is satisfied, $E \uparrow$ makes domestic goods less competitive, which **decreases** NX.
  - **The Income Effect:**
    - Because the higher interest rate reduced activity → $Y \downarrow$.
      - This causes a drop in imports, which **increases** NX.

The Graph of the Model

The standard IS-LM-UIP model is represented by **two side-by-side panels**:

- **Left panel** (Y vs i): downward-sloping IS and horizontal LM intersect at point A, determining $(Y, \bar{i})$.
- **Right panel** (E vs i): the positively sloped UIP line; projecting $\bar{i}$ horizontally to the UIP line determines the equilibrium exchange rate E.

Example 1 — Contractionary Monetary Policy (CB sells bonds, $i \uparrow$)

Look at the previous image!!!

The CB raises $\bar{i}$. IS and UIP are **unchanged** (neither shifts).

- LM shifts **up** → new equilibrium at higher i, lower Y.
- On the UIP panel: we move **along** the UIP to a higher E (appreciation).

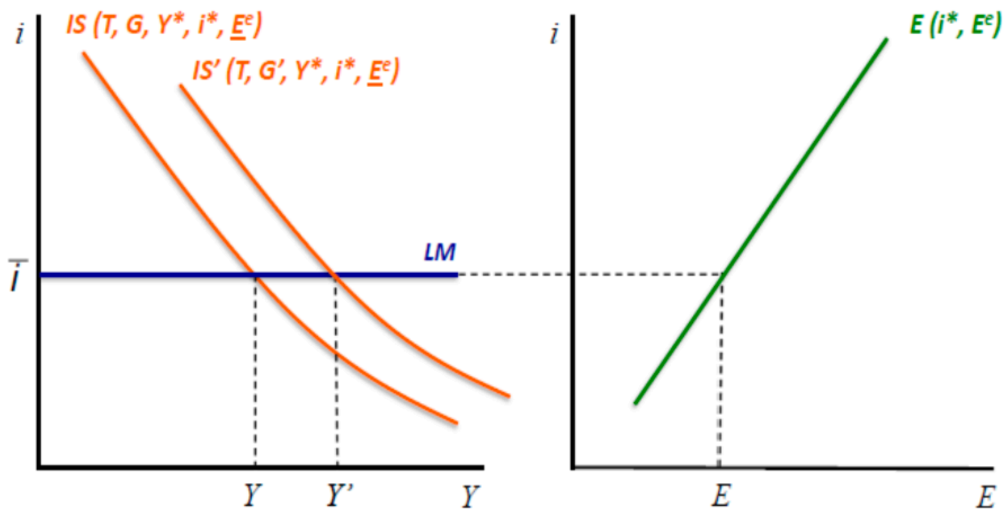
Effects on macro variables:

- $i \uparrow$ (CB decision)
- $Y \downarrow$ (via both I and NX channels)
- $C \downarrow$ (because $Y \downarrow$ with $T = \bar{T}$)
- $I \downarrow$ (because $i \uparrow$ and $Y \downarrow$)
- $E \uparrow$ (appreciation)
- NX: **uncertain** → two opposing forces:

- $Y \downarrow \Rightarrow IM \downarrow \Rightarrow NX \uparrow$ → income effect
- less income/outcome → decrease in IM
- $E \uparrow \Rightarrow X \downarrow, IM \uparrow \Rightarrow NX \downarrow$ → appreciation effect

Example 2 — Expansionary Fiscal Policy ($G \uparrow$, balanced budget: $T \uparrow$ too)

Scenario A: Y was below potential ($Y < Y_n$)



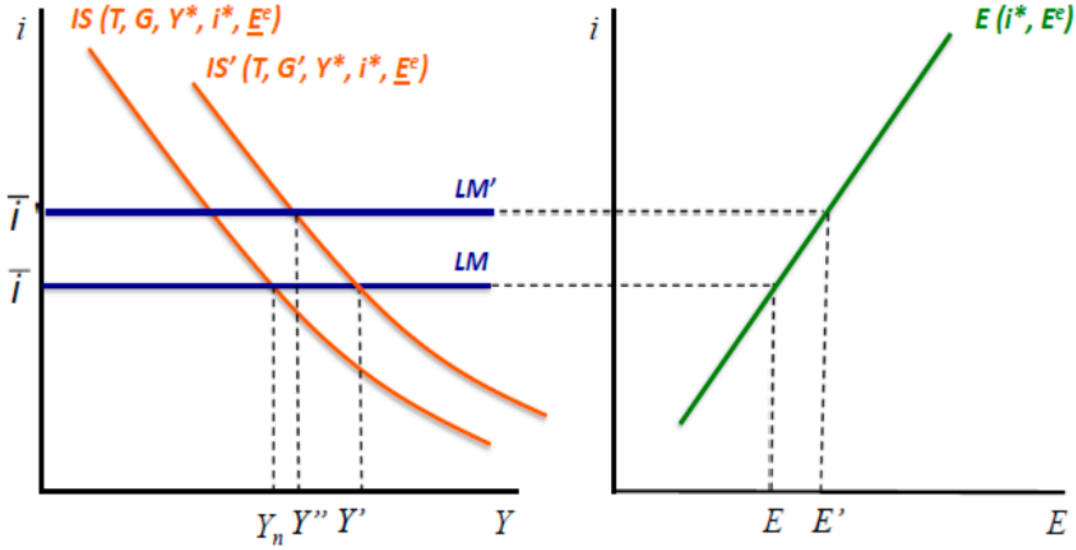
Before the fiscal expansion, $Y < Y_n$, so the CB has no inflation concerns and keeps $\bar{i}$ unchanged. LM and UIP are **unchanged**.

- IS shifts **right** → new equilibrium at higher Y, same i.
- Since i unchanged, E unchanged (we stay at the same point on the UIP).

Effects:

- $Y \uparrow$, $i = \bar{i}$ (unchanged), $E = \bar{E}$ (unchanged)
- $C \uparrow$ (because $Y \uparrow$)
- $I \uparrow$ (because $Y \uparrow$ and i unchanged)
- $G \uparrow$ (by assumption)
- $NX \downarrow$ (because $Y \uparrow \Rightarrow IM \uparrow$, while E and Y^* are unchanged $\Rightarrow X = \bar{X}$)

Scenario B: Y was at or above potential ($Y \approx Y_n$)



The fiscal expansion is motivated by political reasons or an emergency. $G \uparrow$ shifts IS right; without CB intervention Y would overshoot Y_n. The CB raises $\bar{i}$ to bring Y back toward Y_n.

- IS shifts **right**; LM shifts **up**.
- Final equilibrium: $Y \approx Y_n$ (unchanged), but $i \uparrow$ and, via UIP, $E \uparrow$ (appreciation).

Effects:

- $Y \approx \bar{Y}$ (almost unchanged)
- $i \uparrow$, $E \uparrow$ (appreciation)
- $C \approx \bar{C}$ (unchanged, since Y unchanged)
- $I \downarrow$ (because $i \uparrow$ with Y constant)
- $G \uparrow$ (by assumption)
- $NX \downarrow$ (because $E \uparrow \Rightarrow X \downarrow, IM \uparrow$ via M-L; and Y is roughly constant)

Components of demand in the open economy: $C,\ I,\ G,\ NX$ — all four are affected by changes in $Y, Y^*, E, i, i^*, \bar{E}^e$.

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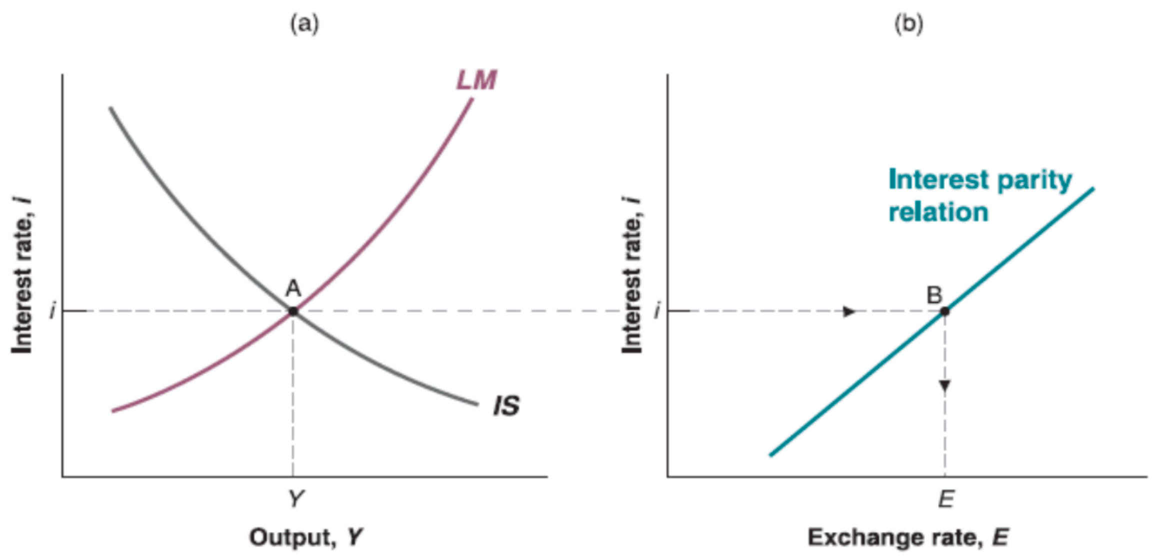
5. 📐 The Non-Standard IS-LM-UIP Model



The non-standard version replaces the fixed-rate LM with the **money market equilibrium**:

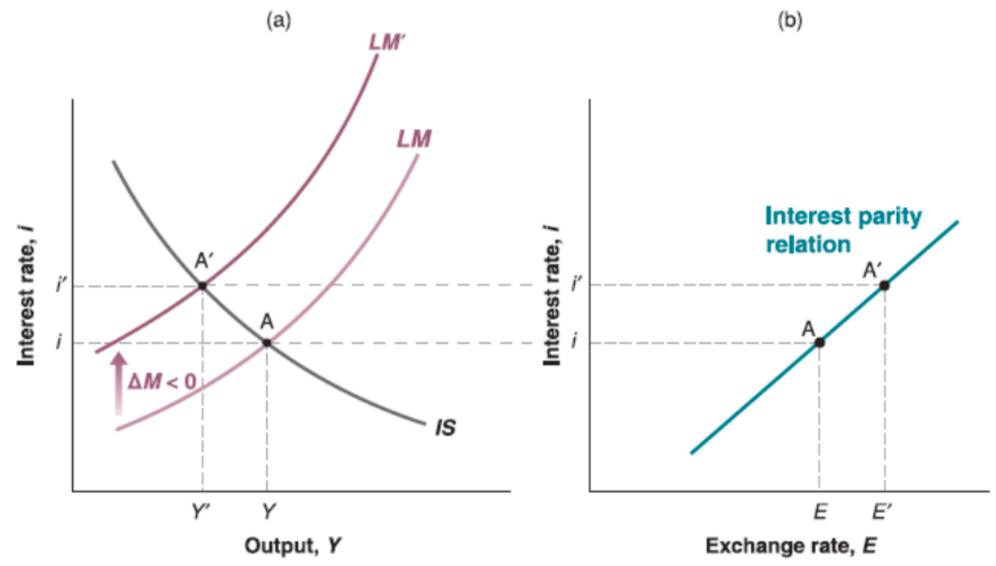
$$\frac{M}{P} = Y \cdot L(i)$$

This is the **same LM as in the closed-economy non-standard model**: **upward sloping** in (Y, i) space. The IS remains identical to the standard version.

The full system is:

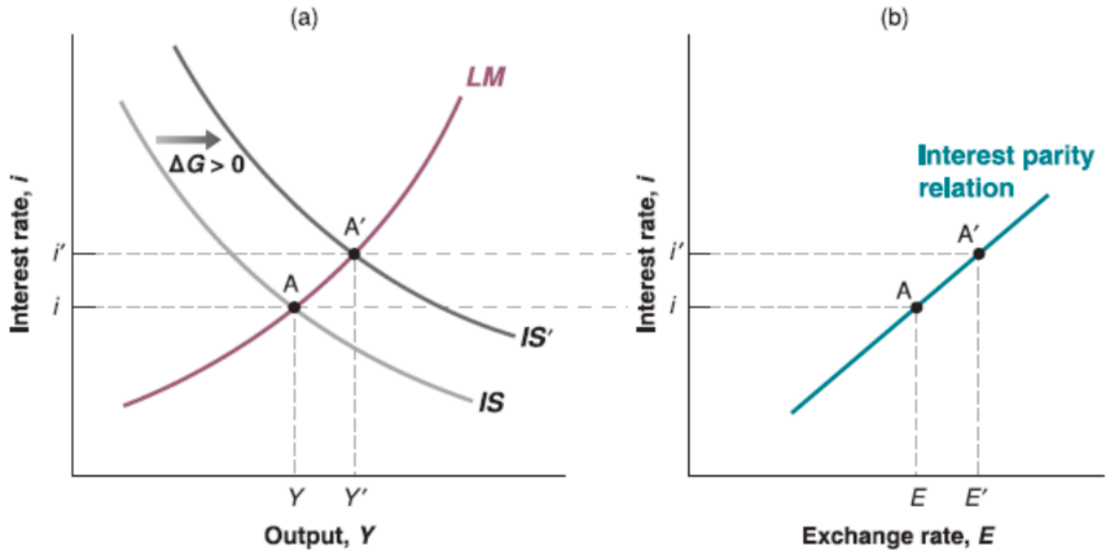
$$IS: \quad Y = C(Y - T) + I(Y, i) + G + NX\left(Y, Y^*, \frac{1+i}{1+i^*}\bar{E}^e\right) LM: \quad \frac{M}{P} = Y \cdot L(i)$$

Monetary Policy — Contraction ($\Delta M < 0$)



- LM shifts **up** (money supply falls → interest rate rises at any output level).
- IS unchanged and UIP unchanged.
- Equilibrium: $i\uparrow$, $Y\downarrow$, and, via UIP, $E\uparrow$ (appreciation).
- **Effect on NX**: **uncertain** (same logic as Scenario A above).

Fiscal Policy — Expansion ($\Delta G > 0$, Y below potential)



- IS shifts **right**.
- LM unchanged (CB does not adjust M).

- But with an upward-sloping LM, the increase in Y raises money demand $\Rightarrow i \uparrow \Rightarrow E \uparrow$ (appreciation).
- Equilibrium: $Y \uparrow$, $i \uparrow$, $E \uparrow$.
- Effects: $G \uparrow$, $C \uparrow$, I ambiguous ($Y \uparrow$ but $i \uparrow$), $NX \downarrow$.
- The budget deficit and the trade balance deficit appear simultaneously: this is the **twin deficit** phenomenon.

> $G \uparrow \Rightarrow Z \uparrow \Rightarrow Y \uparrow \Rightarrow M^d \uparrow \Rightarrow i \uparrow \Rightarrow E \uparrow$

> Budget deficit + Trade deficit = **Twin deficit**

Fiscal Policy — Expansion (Y close to potential)

Same logic as Scenario B in the standard model:

- CB raises $\bar{i}$ (or, in non-standard, the endogenous money market raises i).
- The IS shift is partially offset.
- Y increases by less, but E appreciates, so NX falls.
- The deterioration in NX is **smaller** than in the case where the interest rate does not rise, because the smaller increase in Y means a smaller rise in imports (even though appreciation also hurts exports).

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6. 🌍 Exchange Rate Regimes

Flexible Exchange Rate

- No explicit exchange rate target.
- The CB sets the interest rate (or money supply) freely
  - The **exchange rate adjusts endogenously in the foreign exchange market**.
  - Examples: US, UK, Japan, Canada.

Fixed Exchange Rate

- The country commits to maintaining $E = \bar{E}$ against a foreign currency (or a basket).

- The peg is **maintained by the CB intervening in the foreign exchange market**.
- Euro
  - internally fixed among various members
  - externally (ex. with US,...) flexible

Key implication — domestic interest rate is tied to foreign interest rate:

If $E_t = \bar{E}$ and markets believe the peg will hold, then $E^e_{t+1} = \bar{E}$ too. Substituting into UIP:

$$(1 + i_t) = (1 + i^*_t)\frac{\bar{E}}{\bar{E}} = (1 + i^*_t)$$

Therefore:

> $i_t = i^*_t$

Under a **fixed exchange rate and perfect capital mobility**, the **domestic interest rate must equal the foreign interest rate**.

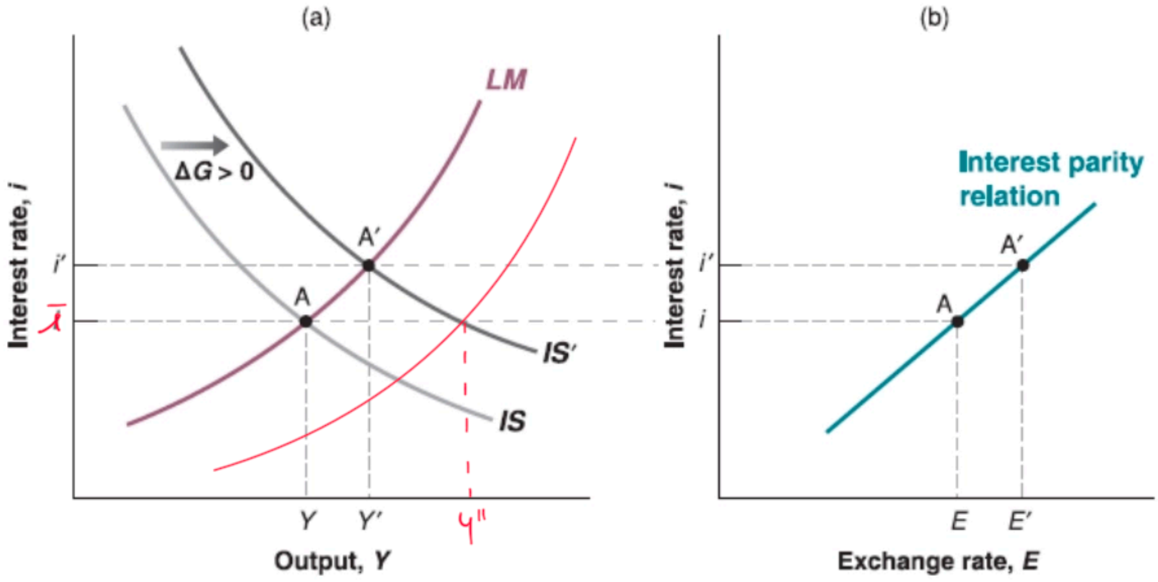
- The **CB gives up monetary policy as an independent instrument**
  - it must track i^* at all times.

Standard model with fixed rates: the LM becomes $i = i^* = \bar{i}$.

Monetary Policy Under Fixed Rates: Ineffective

- The CB cannot lower (or raise) i independently of i^*.
- If it tries to expand by buying bonds, the resulting fall in i triggers capital outflows, depreciation pressure, and the CB must immediately sell foreign reserves to defend the peg, undoing the expansion.
  - Monetary policy cannot be used.

Fiscal Policy Under Fixed Rates: More Powerful



- With i pinned at i^*, a fiscal expansion ($G \uparrow$) shifts IS right **and** the interest rate cannot rise (in the standard model, LM is horizontal at i^*).
- In the non-standard model, the CB **must** accommodate, increasing money supply to keep $i = i^*$, shifting LM down simultaneously with the IS shift.
- **The result:** output increases more than under flexible rates, with no appreciation and no crowding out through the exchange rate.

💡 **Comparison of fiscal policy power:**

- **Flexible rates**, CB raises i: IS shifts right, LM shifts up → Y increases moderately, $E \uparrow$, $NX \downarrow$.
- **Fixed rates:** IS shifts right, LM must shift down (accommodating) → Y increases **more**, E unchanged, **no exchange-rate crowding out**.

Other Exchange Rate Arrangements

- **Crawling peg:**
  - Countries with higher inflation than the US (or anchor country) would experience a steady real appreciation if they fixed E nominally.
  - To avoid this, they allow a **predetermined rate of nominal depreciation,** they "crawl" against the anchor currency, keeping the real exchange rate $\varepsilon = EP/P^*$ approximately stable.

- **Bands / bilateral exchange rates (EMS):**
  - Countries agree to keep exchange rates within a **band** around a central parity.
  - Changes to the central parity require collective agreement.
  - The **European Monetary System (EMS)** operated this way from 1978 to 1998, with Germany's Bundesbank effectively setting the reference interest rate. Major crises (1992–93) caused Italy and the UK to exit. From 1999, the most integrated countries adopted a common currency, the **euro**.

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7. 📎 Appendix — Fixed Exchange Rates, Interest Rates and Capital Mobility

Balance Sheet of the Central Bank

In an open economy, the CB holds two types of assets:

- **Domestic bonds**
- **Foreign exchange reserves** (foreign currency or foreign interest-bearing assets)

Its liability is the **monetary base** (central bank money). The CB can change the monetary base by buying/selling bonds (open market operations) *or* by buying/selling foreign currency (foreign exchange interventions).

Scenario 1 — Perfect Capital Mobility + Fixed Exchange Rate

Assume $i = i^*$ initially.

Suppose the CB attempts an **expansionary open market operation**: it **buys domestic bonds** ($\Delta B > 0$), **expanding the monetary base**.

1. Monetary base $\uparrow$ → $i \downarrow$ below i^*.
2. Investors immediately prefer foreign bonds → capital outflows → sell domestic currency.
3. Domestic currency depreciates → threatens the fixed exchange rate $\bar{E}$.
   - it cannot depreciate since the rate is fixed

4. CB **must** intervene: sells foreign currency ($-\Delta B$) to buy domestic currency → monetary base falls back.
5. **Net result:** monetary base unchanged, i unchanged (back to i^*), only the *composition* of the CB balance sheet changes (more bonds, fewer reserves).

> **Conclusion:**
> Under perfect capital mobility and fixed exchange rates, monetary policy is **completely ineffective**, any open market operation is immediately undone by the required foreign exchange intervention. The CB cannot independently control the interest rate.

Scenario 2 — Imperfect Capital Mobility + Fixed Exchange Rate

If it takes **time** for investors to switch from domestic to foreign bonds (**imperfect capital mobility**), the CB can maintain a lower interest rate for some period:

1. CB buys bonds → $i \downarrow$.
2. **Investors shift to foreign bonds *slowly*** → gradual capital outflows → gradual depreciation pressure.
3. CB sells foreign reserves to defend the peg, but **less urgently** than in Scenario 1.
4. During the transition, the **CB can sustain $i < i^*$ for weeks or months.**
5. Eventually, monetary base returns toward its pre-operation level; the CB suffers a **loss in foreign exchange reserves**.

> **Conclusion:**
> With imperfect capital mobility, the **CB has *some* freedom to deviate from i^*, but only temporarily and at the cost of reserve losses**. The more extensive the capital controls, the greater this freedom.

Scenario 3 — Very High Capital Controls

If domestic investors are legally or practically **unable** to shift into foreign bonds (**strict capital controls**), the CB's open market operation creates only a small depreciation pressure. The required foreign exchange intervention is minimal. The **CB can effectively use monetary policy for a sustained period,** until controls are lifted or reserves become critically low.

Degree of Freedom Under Fixed Rates

The CB's ability to deviate from i^* while maintaining the peg depends on:

1. **Development of financial markets** and investor willingness/ability to shift between assets.
2. **Degree of capital controls** — stricter controls mean more room to manoeuvre.
3. **Level of foreign exchange reserves** — higher reserves allow the CB to sustain more losses without abandoning the peg.

🗂️ Recap Table

Concept	Key formula / result	Remark
Open-economy IS	$Y = C(Y-T) + I(Y,i) + G + NX(Y,Y^*,E)$	Depends on both i and E
UIP (simplified)	$E = \frac{1+i}{1+i^*}\bar{E}^e$	E rises with i; falls with i^*
UIP slope (in i vs E)	$\frac{1+i^*}{\bar{E}^e}$	UIP rotates (doesn't shift) with i^* or $\bar{E}^e$
Standard LM	$i = \bar{i}$	Horizontal; CB sets the rate
Non-standard LM	$M/P = Y \cdot L(i)$	Upward sloping
Two effects of $i \uparrow$ on IS	$I \downarrow$ (direct) + $E \uparrow \Rightarrow NX \downarrow$ (indirect)	IS still slopes down even if I exogenous
Monetary contraction (flexible	$i \uparrow$, $Y \downarrow$, $E \uparrow$, NX uncertain	Move along UIP
Fiscal expansion, $Y < Y_n$ (flexible)	$Y \uparrow$, i unchanged, E unchanged, $NX \downarrow$	Standard scenario A
Fiscal expansion, $Y = Y_n$ (flexible)	$Y \approx$ const, $i \uparrow$, $E \uparrow$, $NX \downarrow$	Scenario B: CB reacts
Twin deficit (non-standard, $G \uparrow$)	Budget deficit + trade deficit simultaneously	$G \uparrow \Rightarrow Y \uparrow \Rightarrow i \uparrow \Rightarrow E \uparrow \Rightarrow NX \downarrow$
Fixed exchange rate	$E = \bar{E} \Rightarrow i = i^*$	CB gives up monetary policy
Fiscal policy, fixed rates (non-std)	More powerful: LM accommodates, no E appreciation	CB must increase M to keep $i = i^*$

Concept	Key formula / result	Remark
Perfect capital mobility + fixed ER	Monetary policy completely ineffective	Open market operations immediately reversed by FX intervention
Imperfect capital mobility + fixed ER	Some temporary room for $i \neq i^*$	At cost of reserve losses; depends on controls and reserve level

🌐

Chapter 21 - Exchange rate regimes

☑ Reviewed	☐

Reference: Lectures and Chapter 21 Textbook (pp. 444–465, Appendix 2 excluded)

Context: Extensions – The Open Economy. We now move beyond the short-run IS-LM-UIP model of Chapter 20 to ask: does the choice of exchange rate regime matter in the medium run? And what are the risks and trade-offs associated with each regime?

> 💡 **Key message:** In the short run, fixed and flexible exchange rates produce sharply different outcomes. In the medium run, however, the real exchange rate adjusts through prices even under a fixed nominal rate, so the two regimes converge to the same output level. Yet the path of adjustment under fixed rates is slow and painful, crises are possible, and countries must accept the trilemma: they can only have two of the three — fixed exchange rates, perfect capital mobility, and independent monetary policy.

---

1. 📐 The Real Exchange Rate

The **real exchange rate** is defined as:

$$\varepsilon = \frac{EP}{P^*}$$

where E is the nominal exchange rate, P is the domestic price level, and P^* is the foreign price level.

This single formula is the key to understanding the difference between **regimes**:

- Under **flexible exchange rates**:
  - E can change

→ ε can be adjusted **immediately**, even if P and P^* are fixed (short run).

- Under **fixed exchange rates**:
  - $E = \bar{E}$ is constant

    → in the short run, with P and P^* also fixed, ε **cannot change**.

The central insight of Chapter 21 is that this short-run asymmetry disappears in the **medium run**, because prices are no longer fixed:

- In the medium run, even under a fixed nominal exchange rate, P and P^* can move → $\varepsilon = \bar{E}P/P^*$ can still adjust through the **relative price level**.

	Short run	**Medium run**
Flexible vs Fixed	The two regimes behave **differently**	The regime **does not matter**, both reach the same Y and ε
How ε adjusts	Flexible: via E Fixed: cannot adjust	Both: via P relative to P^*
Monetary policy	Effective under flexible; surrendered under fixed	Irrelevant in the long run

---

2. ⚖️ Flexible vs Fixed — Summary of Trade-offs

Before proceeding to the formal analysis, it is useful to keep in mind the broad trade-off:

- **Flexible exchange rates** are volatile (the exchange rate moves with every expectation update), but **preserve monetary policy** and allow **fast adjustment of** ε**.**
- **Fixed exchange rates** give up monetary policy (with perfect capital mobility), but **facilitate trade** by **eliminating exchange rate uncertainty,** and **can import credibility** when domestic CB is untrustworthy.

- The consensus today is for **flexible exchange rates,** but with two important **exceptions**:
  - **common currency areas**
  - situations of **CB credibility failure**

→ (discussed in section 7 below).

3. 📦 The IS Under Fixed Exchange Rates

Under a **fixed exchange rate regime with perfect capital mobility**, two conditions hold simultaneously:

$$E = \bar{E}$$

$$i = i^*$$

The first follows by **definition** (the peg); the second from the **UIP** condition:

- if E is expected to remain at $\bar{E}$ forever, the interest parity collapses to $i = i^*$

Starting from the general goods-market equilibrium:

$Y = C(Y - T) + I(Y, r) + G + NX(Y, Y^*, \varepsilon)$

and substituting $r = i - \pi^e$, $i = i^*$, and $\varepsilon = \bar{E}P/P^*$, the IS relation becomes:

$$Y = C(Y - T) + I(Y,\, i^* - \pi^e) + G + NX\left(Y,\, Y^*,\, \frac{\bar{E}P}{P^*}\right)$$

In compact notation:

$$Y = Y\left(\frac{\bar{E}P}{P^*},\, G,\, T,\, i^* - \pi^e,\, Y^*\right)$$

$$(- \quad + \quad - \quad - \quad +)$$

Interpreting the signs:

- $\varepsilon = \bar{E}P/P^* \uparrow$:
  - domestic goods become relatively more expensive → $IM \uparrow$, $X \downarrow$ → $NX \downarrow$ → $Y \downarrow$ **(negative)**
- $\varepsilon \downarrow$:
  - the opposite → $NX \uparrow$ → $Y \uparrow$

- $G \uparrow$: direct demand stimulus (**positive**)
- $T \uparrow$: reduces disposable income → $C \downarrow$ (**negative**)
- $i^* - \pi^e \uparrow$: real cost of borrowing rises → $I \downarrow$ (**negative**)
- $Y^* \uparrow$: foreign demand for domestic exports rises (**positive**)

Note that the CB has no independent role here: the interest rate is pinned at i^*, and the exchange rate is pinned at $\bar{E}$. The only remaining policy levers are **fiscal policy** (G, T) and the possibility of a **one-time devaluation**.

---

4. 🔄 Reasoning on the Medium Run

The problem: a country with a trade deficit and a high ε

Suppose a country is stuck with a **real exchange rate that is too high** (domestic goods are overpriced relative to foreign goods).

This generates a **trade deficit** and **output below potential** $(Y < Y_n)$.

Short-run comparison:

- **Flexible exchange rates**:
  - the CB can lower i → $E \downarrow$ (nominal depreciation) → with P, P^* fixed, $\varepsilon \downarrow$ → Marshall-Lerner: $X \uparrow, IM \downarrow, NX \uparrow$ → $Y \uparrow$.
  - Problem solved quickly.
- **Fixed exchange rates**:
  - $i = i^*$ is pinned
  - $E = \bar{E}$ cannot move
  - ε cannot fall
  - The trade deficit and recession **persist in the short run**.

The medium-run mechanism: prices adjust

In the medium run, prices are **no longer fixed**. Their behaviour is described by the Phillips Curve:

$$\pi - \pi^e = \frac{\alpha}{L}(Y - Y_n)$$

Assuming constant expected inflation $\pi^e = \bar{\pi}$ (and likewise $\pi^* = \bar{\pi}$ when the foreign economy is at potential):

$$\pi - \bar{\pi} = \frac{\alpha}{L}(Y - Y_n)$$

Now follow the adjustment path under **fixed exchange rates**:

1. Start:
   - $Y = Y_n$
   - $\pi = \pi^* = \bar{\pi}$
   - $\rightarrow \varepsilon = \bar{E}P/P^*$ is at some high level causing a deficit.
2. Because $Y < Y_n$, the PC implies $\pi < \bar{\pi}$, so $\pi < \pi^*$.
   - **Domestic prices rise more slowly** than foreign prices.
3. With $E = \bar{E}$ fixed: $\varepsilon = \bar{E}P/P^* \downarrow$ (**domestic goods become relatively cheaper** over time).
4. Marshall-Lerner: $NX \uparrow \rightarrow Y \uparrow \rightarrow$ output gradually returns to Y_n.
5. Once $Y = Y_n$: $\pi = \bar{\pi} = \pi^*$ again $\rightarrow \varepsilon$ is constant at its new, lower equilibrium level.

Summary of the distinction:

- **Short run**: a fixed nominal exchange rate $\Rightarrow$ a fixed real exchange rate.
- **Medium run**: **NO** $\rightarrow \varepsilon$ can still adjust through **relative prices**, even under fixed E.

> ⚠️ The medium-run adjustment through prices is **slow and painful**:
> → output remains below potential and unemployment remains high for an extended period.

Alternative adjustments under fixed rates

Rather than waiting for the slow price adjustment, two faster alternatives exist:

1. One-time devaluation:
   - lower E directly $\rightarrow$ immediate $\varepsilon \downarrow \rightarrow$ real depreciation $\rightarrow NX \uparrow, Y \uparrow$.
     - CB changes the fixed rated $\rightarrow$ new peg
   - Can work even in the short run

- **But must not be abused**
  - repeated devaluations trigger exchange rate crises (see section 5).

2. **Float**:
   - **abandon the peg** and move to a flexible exchange rate, allowing the CB to use monetary policy freely.

---

5. 💥 Exchange Rate Crises Under Fixed Exchange Rates

Why crises can arise

Under a fixed exchange rate system, **investors could begin to expect an exchange rate adjustment**, either a devaluation or a decision to float. This can happen because:

- The **real exchange rate is too high** (domestic currency overvalued), often because the **domestic inflation rate has persistently exceeded the foreign one**
  - this happens when a country pegs its nominal exchange rate to the currency of a lower-inflation country
  - the resulting steady real appreciation makes the trade position worse over time, and financial markets begin to anticipate a devaluation.
- **Internal conditions** (e.g., a recession) **call for a lower interest rate**, which cannot be achieved under the fixed peg but could be achieved if the exchange rate were allowed to float.

The UIP and devaluation expectations

Recall the UIP condition:

$$i_t = i_t^* - \frac{E_{t+1}^e - E_t}{E_t}$$

- Under normal fixed-rate conditions: $E_t = E_{t+1}^e = \bar{E}$, so the last term is zero and $i_t = i_t^*$.
- Now suppose **investors begin to expect a devaluation**: $E_{t+1}^e < E_t$ (they expect the domestic currency to be worth less next period).

- The term $(E_{t+1}^e - E_t)/E_t$ becomes **negative**, so i_t must rise **above** i_t^* to compensate investors for the expected loss on domestic assets.

For example: if markets believe there is a 25% chance of a 20% devaluation next month, the expected depreciation is $0.25 \times (-20\%) = -5\%$. Investors will only hold domestic bonds if i exceeds i^* by 5% per month, roughly **60% annualised**.

What can the government and CB do?

Faced with devaluation expectations, authorities have a limited menu:

1. **Announcements** → try to **convince markets** that devaluation is not coming.
   - Rarely effective on its own; words are cheap.
2. **Raise i** → to **satisfy (or approach) the UIP** condition and make domestic bonds attractive:
   - If the interest rate increase is **too small** (insufficient to satisfy UIP), investors still prefer foreign bonds → **capital outflows** → they sell domestic currency in the FX market → depreciation pressure.
     - Mathematically, this is explained by the Uncovered Interest Parity (UIP) condition $E_t = \frac{1+i_t}{1+i_t^*} E_{t+1}^e$:
       - Here is how the math plays out:
         1. Investors expect a devaluation, meaning the expected future exchange rate (E_{t+1}^e) decreases.
         2. According to the formula, if E_{t+1}^e drops, the current exchange rate (E_t) must also drop (depreciate) unless the central bank raises the domestic interest rate (i_t) enough to offset it.
         3. **If the central bank's increase in i_t is too small, the math dictates that $E_t$must fall.**
            → This mathematical imbalance is the "pressure" causing the currency's value to drop.
   - The CB must **buy domestic currency / sell foreign reserves** to defend the peg, this depletes reserves rapidly.
     - since investors sold domestic currency, there is an oversupply of it, being E fixed, CB must absorb the oversupply by buying domestic

currency.

- High i depresses domestic demand: $I \downarrow$ → recession worsens. This course makes sense **only if the required Δi is small and temporary**.

3. **Give in** → devalue or float.
    - This **validates markets' expectations** and can occur even if the original belief was unfounded:
        - it is a **self-fulfilling expectation**.
            - When investors expect a devaluation, they sell off the domestic currency. To defend the fixed exchange rate, the Central Bank is forced to drastically raise interest rates to convince people to hold domestic bonds.
            - Because high interest rates hurt the domestic economy, the Central Bank often decides the pain is too great and "gives in" by devaluing the currency. In the end, the investors' fear essentially forces the exact event they predicted to happen.

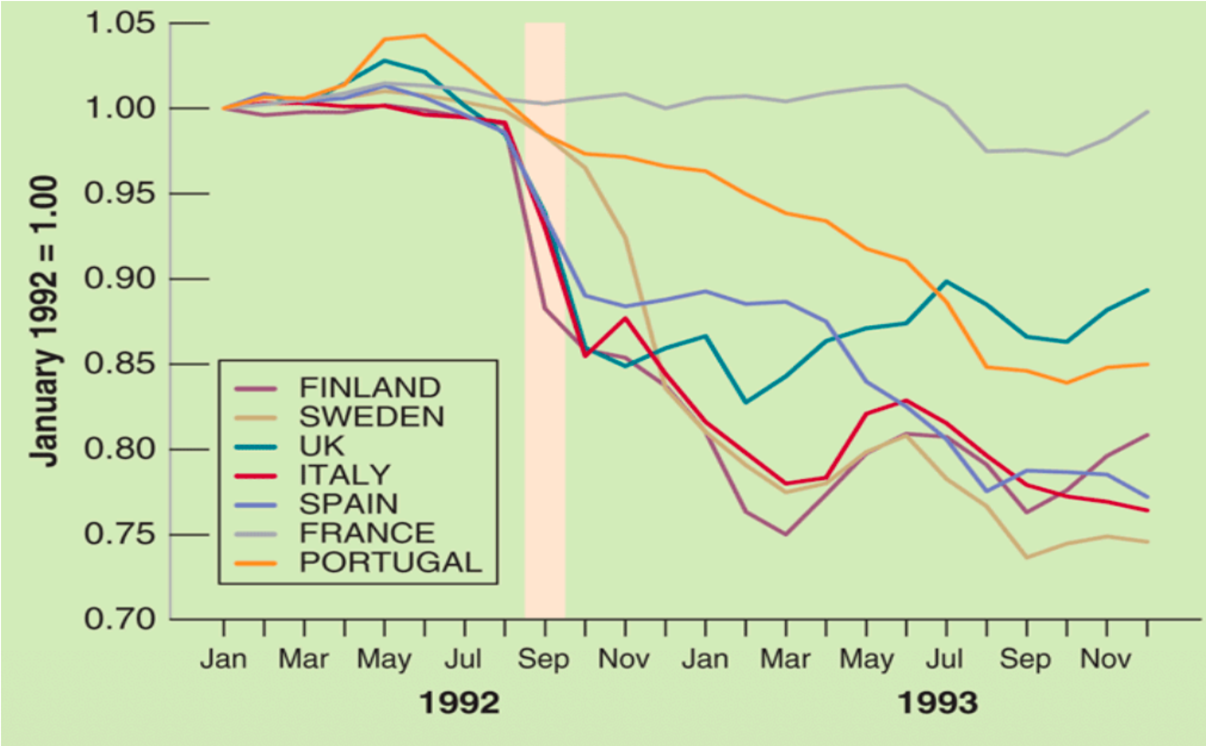
> ⚠️ A devaluation can happen simply because people believe it will happen, the crisis itself forces the outcome that triggered the crisis.

- important factors
    - Perceived probability of devaluation
    - Amount of foreign currency in CB balance sheet
- Expectations that a devaluation may be coming can trigger an exchange rate crisis
    - As previously said faced with the expectations of the valuation, the government has two options:
        - give in and devaluate
        - fight to maintain parity → high interest rate

Focus: The 1992 EMS Crisis

- The European Monetary System (**EMS**), established in **1979**, was a fixed-parity-with-bands arrangement.

    - **Each country had to maintain its exchange rate vis-à-vis all other members with a narrow band**
- After years of relative stability (only two realignments between 1987 and 1992), speculation mounted in **mid-1992** due to the **macroeconomic consequences of German reunification**:
    - **reunification boosted German demand**
        - **Bundesbank raised interest rates to contain inflation**
        - other EMS members needed *lower* rates to fight unemployment
            - BUT they were **forced to match German rates** just to maintain their parities
- In **September 1992**, speculative attacks erupted.
    - Countries followed different paths:
        - Spain devalued
        - Italy and the UK **suspended EMS participation** (floated)
        - France raised rates and toughed it out
    - By November 1992 a new wave of speculation hit Portugal and Sweden.
    - In July 1993, EMS countries adopted wider bands (±15%), effectively allowing much more flexibility.
- The system ran in this looser form until the euro was adopted in 1999.



Exchange rates of European countries relative to the Deutsch mark, January 1992 to December 1993

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6. 📈 Exchange Rates Under Flexible Exchange Rates

Why the exchange rate is complex and volatile

In Chapter 20 we treated the UIP as a simple link: $E_t = \frac{1+i_t}{1+i_t^*} E_{t+1}^e$.

But E_{t+1}^e is itself not fixed → it depends on expectations about E_{t+2}, and so on.

Substituting forward repeatedly:

$$E_t = \frac{(1+i_t)(1+i_{t+1}^e)\cdots(1+i_{t+n}^e)}{(1+i_t^*)(1+i_{t+1}^{*e})\cdots(1+i_{t+n}^{*e})} \cdot E_{t+n+1}^e$$

This expression reveals three important implications:

1. **E_t moves one-for-one with the far-future expected exchange rate E_{t+n+1}^e:**
   - any news about the long-run equilibrium value of the exchange rate (e.g., news about future current account balances) immediately affects today's E.

2. **E_t moves whenever *current or expected future interest rates* change in either country**:
   - a monetary policy announcement that shifts expectations about the path of i over the next several years has an immediate, potentially large effect on E_t.
3. **→ E_t is therefore inherently volatile**:
   - since it aggregates a long chain of uncertain expectations, any new piece of information (about monetary policy, fiscal policy, output, inflation, geopolitics) can and will move E_t.

- The upshot is that the **relation between the interest rate and the exchange rate is not mechanical.**
  - Whether a cut in i leads to a large or small depreciation depends entirely on how markets update their expectations about the future path of interest rates in both countries.
  - **A country operating under flexible exchange rates must accept substantial exchange rate fluctuations as the price of monetary independence**.

The "Dance of the Dollar" (1980–1990)

- The "dance of the dollar" refers to the massive appreciation and subsequent depreciation of the US dollar during the 1980s. It serves as a primary example of how flexible exchange rates do not move mechanically but are instead driven by shifts in policy and market expectations.

1. **The Core Mechanism: Expectations**
   Under flexible exchange rates, the current exchange rate (E_t) is not just a reflection of today's interest rates.
   - **Forward-Looking:** E_t depends on the entire expected future path of domestic vs. foreign interest rates (i vs. i^*) and the long-run expected exchange rate.
   - **Volatility:** Because E_t aggregates a long chain of uncertain expectations, any news about future monetary or fiscal policy causes

immediate and often large fluctuations.

2. **Phase 1: Rapid Appreciation (1980–1985)**
   The dollar saw a sharp rise in value due to a specific "policy mix" in the US:
   - **Tight Monetary Policy (Volcker Disinflation):**
     - The **Fed drastically raised interest rates to fight high inflation**.
   - **Expansionary Fiscal Policy (Reaganomics):**
     - Large tax cuts and increased spending further pushed up both short-term and long-term interest rates.
   - **The Result:** High US interest rates relative to the rest of the world attracted massive capital inflows, causing the dollar to appreciate and creating a large current account deficit ($CA < 0$).
3. **Phase 2: The Reversal (1985–1990)**
   By 1985, the dollar was considered overvalued, leading to a coordinated international effort to bring it down:
   - **The Plaza Accords (1985):** Major economies (G5) agreed to intervene in currency markets to **move the dollar lower**.
   - **Policy Shift:** A coordinated reversal of the previous years' policies led to a steady depreciation of the dollar back toward its 1980 levels.
4. **Key Lessons for Flexible Regimes**
   - **Not Mechanical:**
     - A shift in interest rates only leads to a **large depreciation if markets believe it represents a permanent or long-term policy shift**.
   - **Price of Independence:**
     - While flexible rates allow a country to maintain an independent monetary policy, the "price" is exposure to substantial and sometimes erratic exchange rate volatility.

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7. 🌍 Choosing Between Exchange Rate Regimes

The general consensus

The mainstream view today is that **flexible exchange rates are preferable**, for two main reasons:

a. In the short run, they **preserve monetary policy and the exchange rate** as **independent adjustment tools**.

b. They **avoid the risk of exchange rate crises** and the associated **misalignment between domestic needs and foreign interest rates.** (ex. EMS 1992)

However, there are **two important exceptions** where fixed rates (or a common currency) may be preferred:

Exception 1 — Common Currency Areas (CCA)

When countries are **tightly integrated**, the costs of giving up exchange rate flexibility are low and the benefits of eliminating transaction costs and exchange rate uncertainty are high.

Robert Mundell identified the conditions for an **optimal currency area**: it must satisfy **at least one** of the following:

a. Countries experience **similar shocks**
   - → they would have chosen similar monetary policies anyway, so sharing one policy is no constraint.

b. If shocks differ, **prices and wages must be flexible**
   - → real exchange rate adjustment occurs through internal price movements.

c. If shocks differ, **factor mobility must be high**
   - → workers move from depressed to thriving regions, substituting for the missing exchange rate instrument.

	Similar shocks	High factor mobility
United States	Partially (states differ)	✅ Yes — high labour mobility
Euro area	❌ No — very heterogeneous	❌ No — limited due to language/culture

- The US satisfies the third condition well and is close to an optimal Common Currency Area.
- Europe satisfies **neither condition fully**, which is why adjustment within the euro area after asymmetric shocks (e.g., the 2010–12 sovereign debt crisis) has been slow and painful.

Exception 2 — Hard Pegs and CB Credibility

If a central bank **cannot be trusted** to pursue a responsible monetary policy (e.g., because it has historically monetised fiscal deficits, producing hyperinflation), fixing the exchange rate can **import credibility** from the anchor country:

- Under a fixed rate, the CB must keep $i = i^*$
- The most common mechanism is a **hard peg**, of which there are two forms:

  a. **Currency board**:

     - A monetary authority stands ready to **convert domestic currency into foreign currency at a fixed rate.**
     - It **holds foreign reserves equal to the monetary base** and **cannot conduct open market operations** (it cannot create money independently).
       - Examples: Hong Kong, Bulgaria, Argentina 1991–2001.

  b. **Dollarisation**:

     - The country **replaces its domestic currency entirely** with a foreign currency (usually the US dollar).
     - There is no longer a domestic monetary policy.
     - This is the most extreme and most credible commitment.

> ⚠️ Hard pegs must be accompanied by **fiscal discipline**.
> A fixed exchange rate alone will not convince markets if large budget deficits continue to threaten future money creation.

The Trilemma

The three objectives (**fixed exchange rates**, **free capital mobility**, **independent monetary policy**) are **mutually incompatible**. A country can have at most **two out of three**:

$$\text{Fixed exchange rate} + \text{Perfect capital mobility} \Rightarrow \text{no monetary policy}$$

$$\text{Fixed exchange rate} + \text{Monetary policy} \Rightarrow \text{no perfect capital mobility}$$

$$\text{Monetary policy} + \text{Perfect capital mobility} \Rightarrow \text{no fixed exchange rate}$$

The **trilemma** (also called the **impossible trinity**) explains every major exchange rate arrangement in the world:

- **Flexible ER + open capital account** (US, UK, Japan): monetary policy is sovereign; no fixed rate.
- **Fixed ER + open capital account** (euro area, currency boards): monetary policy is surrendered.
- **Fixed ER + monetary policy** (China historically): requires capital controls.

Political and economic goals determine which two the country chooses to keep.

8. 🇦🇷 Argentinian currency board (1991-2002)

Argentina provides a classic example of using a **hard peg** to "import" credibility and the eventual risks of being unable to adjust the exchange rate during a crisis.

1. **The Context: Hyperinflation**
   Before 1991, Argentina suffered from **hyperinflation** (2,300% in 1990) and persistent budget deficits. The government had historically financed debt through money creation, destroying the central bank's credibility.
2. **The Solution: A Currency Board**
   In 1991, the country established a **currency board**, which is a form of a hard peg where the monetary authority holds foreign reserves equal to the monetary base.
   - **The Peg:** Fixed the peso to the US dollar at a 1-to-1 ratio.
   - **The Constraint:** The Central Bank could not conduct open market operations (it could not create money independently).
   - **The Result:** By "tying the hands" of the CB, they imported the credibility of the US Federal Reserve.
3. **Phase 1: Success (1991–1999)**
   The arrangement worked effectively for nearly a decade:
   - **Disinflation**: Inflation dropped from 2,300% in 1990 to 4% by 1994.
   - **Growth**: The economy saw strong output growth, averaging 5% between 1991 and 1999.

4. **Phase 2: The Downward Spiral (Late 1990s)**
   The system began to fail due to **external shocks and the rigidities of the fixed rate**:
   - **Real Appreciation:**
     - In the late 90s, the US dollar appreciated. Because the peso was pegged to the dollar, it also appreciated, making **Argentinian exports expensive and reducing demand.**
   - **Recession and Deficit:**
     - The fall in demand led to a **decline in output and a growing budget deficit.**
   - **The Trilemma Trap:**
     - Because they had a fixed rate and capital mobility, they had no independent monetary policy. They **could not lower interest rates** or depreciate the currency to fight the recession.
5. **Phase 3: Collapse and Lessons**
   - **The End:** In 2001, Argentina defaulted on part of its debt. In **2002, the currency board was abandoned**, and the peso was allowed to float, causing it to lose roughly 75% of its value (moving from $E = 1$ to $E = 1/3.75$).
   - **The Cost:** Extremely costly to exchange back dollars in pesos for banks to people who had bought US bonds.
   - **Lessons Learned:**
     - Some economists argue that a currency board should only be a **temporary tool** to break hyperinflation, and countries should move to a floating rate once inflation is under control.
     - Others suggest that if a country chooses a hard peg, they should go all the way to **dollarization** (replacing the currency entirely) to maximize credibility.

🗂️ Recap Table

Concept	Key formula / result	Remark
Real exchange rate	$\varepsilon = \bar{E}P/P^*$	Can adjust via E (flexible) or P/P^* (medium run, both regimes)
IS under fixed ER	$Y = Y(\bar{E}P/P^*, G, T, i^* - \pi^e, Y^*)$	Signs: $(-, +, -, -, +)$
Medium run under fixed ER	$Y < Y_n \Rightarrow \pi < \pi^* \Rightarrow \varepsilon \downarrow \Rightarrow NX \uparrow \Rightarrow Y \uparrow$	Slow and painful; price adjustment via PC
Phillips Curve	$\pi - \bar{\pi} = (\alpha/L)(Y - Y_n)$	Engine of medium-run price adjustment
UIP under devaluation expectations	$i_t = i_t^* - (E_{t+1}^e - E_t)/E_t \neq i^*$	Requires $i \uparrow$ above i^* to defend peg
Exchange rate crisis options	1. Convince markets (rarely works); 2. Raise i (costly); 3. Devalue/float	Devaluation can be self-fulfilling
E_t under flexible rates	$E_t = \prod_{k=0}^{n} \frac{1+i_{t+k}^e}{1+i_{t+k}^{*e}} \cdot E_{t+n+1}^e$	Depends on entire future path of i vs i^*
Optimal currency area (Mundell)	Need: similar shocks, OR flexible wages/prices, OR high factor mobility	US ≈ optimal; EU not fully
Hard peg — currency board	Full reserves; no open market operations	Stronger commitment than soft peg
Hard peg — dollarisation	Domestic currency replaced entirely by foreign one	Most extreme; no domestic monetary policy at all
The trilemma	Fixed ER + Capital mobility + Monetary policy: pick only 2	Fundamental constraint on policy design

📊

Chapter 23 - Fiscal policy: a summing up

☑ Reviewed ☐

Reference: Lectures — Textbook Chapter 23 (pp. 492, 494, pp. 496–500, pp. 504–506)

Context: Extensions — Back to Policy. After analysing fiscal policy in the short run, medium run, and open economy, we now turn to the **dynamics** of government debt over time. The chapter introduces the government budget constraint and uses it to study whether the debt-to-GDP ratio is sustainable or will diverge, and what dangers arise when it becomes too high.

> 💡 **Key message:** The evolution of the debt-to-GDP ratio depends critically on the relationship between the real interest rate r and the GDP growth rate g, as well as on the primary balance d. When $r > g$, debt tends to be self-reinforcing — high debt requires large primary surpluses, and failure to achieve them can trigger a vicious spiral. When $r < g$, even a country running a primary deficit can see its debt ratio stabilise.

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1. 📋 The Government Budget Constraint

The government budget constraint expresses the relationship between debt, spending, taxes, and interest payments. All variables are in **real terms** (inflation-adjusted).

Suppose the government budget is initially balanced. Define:

- G_t = **government spending** on goods and services (excluding transfers)
- T_t = taxes **minus** transfers during year t
  - **net tax revenue**
- B_t = **public debt** at the end of year t (sum of all past deficits)
- B_{t-1} = **public debt** at the beginning of year t
- r = real interest rate (assumed constant)
- rB_{t-1} = **real interest payments** on existing debt

The **budget deficit** in year t is:

$$\text{deficit}_t = rB_{t-1} + G_t - T_t$$

- The term $(G_t - T_t)$ is called the **primary deficit**
  - Its opposite $(T_t - G_t)$ is the **primary surplus**.
  - The primary deficit excludes interest payments and reflects only the structural fiscal position.

Since the **change in debt equals the deficit**, the **government budget constraint** is:

$$B_t - B_{t-1} = rB_{t-1} + G_t - T_t$$

In words: **change in debt = interest payments + primary deficit.**

Rearranging, the **level of debt at the end of year t** is:

$$B_t = (1 + r)B_{t-1} + G_t - T_t$$

- Debt at the end of year t equals $(1 + r)$ times last period's debt plus this year's primary deficit.
- Even with a primary deficit of zero, **existing debt grows at rate r because of interest payments.**

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2. 📈 The Evolution of the Debt-to-GDP Ratio

- To assess fiscal sustainability, it is more meaningful to track **debt relative to GDP** rather than in levels
  - a large country with high GDP can sustain more debt than a small one

Define the **debt-to-GDP ratio**:

$$b_t = B_t/Y_t$$

Dividing the budget constraint by Y_t and multiplying/dividing the first term by Y_{t-1}:

$$\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{Y_{t-1}} \cdot \frac{Y_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

- from: $\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{Y_t} \cdot \frac{Y_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$

Let g be the **growth rate of real output**

- so $Y_t = (1+g)Y_{t-1}$
  - $\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{(1+g)Y_{t-1}} \cdot \frac{Y_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$
  - giving $Y_{t-1}/Y_t = 1/(1+g)$
  - using the approximation $(1+r)/(1+g) \approx 1+r-g$ (Appendix 2, Proposition 6):

> $$\frac{B_t}{Y_t} = (1+r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Solving for the **change** in the debt-to-GDP ratio:

> $$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

- This is the **key equation** of the chapter.
- The **debt ratio increases when:**
  - r **is higher** (debt accumulates faster through interest)
  - g **is lower** (GDP grows more slowly, making existing debt relatively heavier)
  - the **initial debt ratio b_{t-1} is higher**
  - the **primary deficit/GDP ratio is larger**

Intuition:

- r is the growth rate of the stock of debt (through interest)
- g is the growth rate of the denominator (GDP)
- → **If $r > g$, the numerator grows faster than the denominator**
  - the **ratio rises**, even with a zero primary deficit.

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3. 📉 The Debt Ratio in the Long Run *(not in the textbook)*

- To study long-run dynamics, we treat the debt-to-GDP equation as a **first-order difference equation**
  - an equation relating a variable to its own past value.
  - using lowercase for ratios to GDP:

> $$b_t = (1+r-g)\,b_{t-1} + d$$

- where $d = (G_t - T_t)/Y_t$ **is the primary deficit/GDP ratio** (assumed constant)

- This takes the general form $y_t = \beta y_{t-1} + A$, with:
  - Slope $= 1 + r - g$
  - Vertical intercept $= d$
  - Horizontal intercept $= -d/(1 + r - g)$

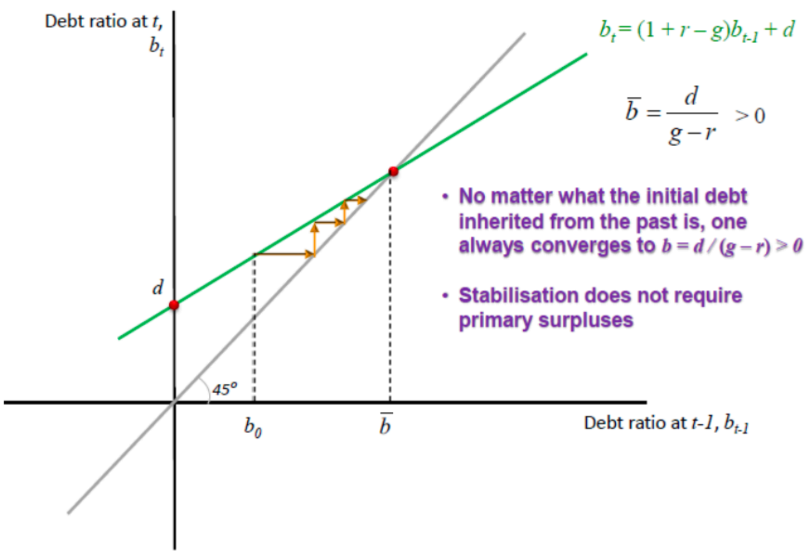
The line is called the **debt-to-GDP line**, plotted in (b_{t-1}, b_t) space.

- Its intersection with the 45° line gives the **steady state** $\bar{b}$, where $\Delta b = 0$:

> $0 = (r - g)\bar{b} + d \quad \Longrightarrow \quad \bar{b} = \dfrac{d}{g - r}$

We analyse **four scenarios** depending on the sign of d and the relative magnitudes of r and g.

📌 Scenario 1 → Primary Deficit ($d > 0$) with $r < g$ → Sustainable (Convergent)



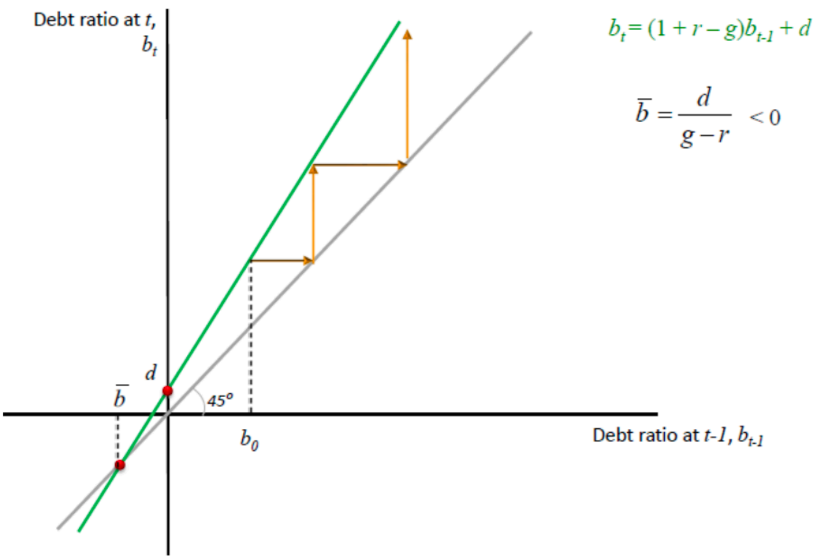
- **Setup:** The government runs a **primary deficit** ($G > T$), but the **economy grows faster than the interest rate.**
- **Debt-to-GDP line:**
  - slope $1 + r - g < 1$ → **flatter than the 45° line**
    - since r < g by hypotheses (economy grows faster han interest rate)
  - vertical intercept $d > 0$; horizontal intercept < 0.
- **Dynamics:** Starting from any positive b_0, the debt ratio increases each period but at a **decreasing rate**
  - it approaches the steady state debt ratio, but at a lower and lower rate
  - it converges to the steady state:

$$\bar{b} = \frac{d}{g - r} > 0$$

- The government is a net debtor, but because the growth rate outpaces the interest rate, this situation is **sustainable**:
  - **debt converges to a finite level relative to GDP.**
  - **no primary surplus is required to stabilise the ratio**.
  -

> ✅ **Special case** ($d = 0$, $g > r$): the steady state is $\bar{b} = 0$
> → the debt ratio converges to zero.

📌 Scenario 2 → Primary Deficit ($d > 0$) with $r > g$ → Unsustainable (Divergent)



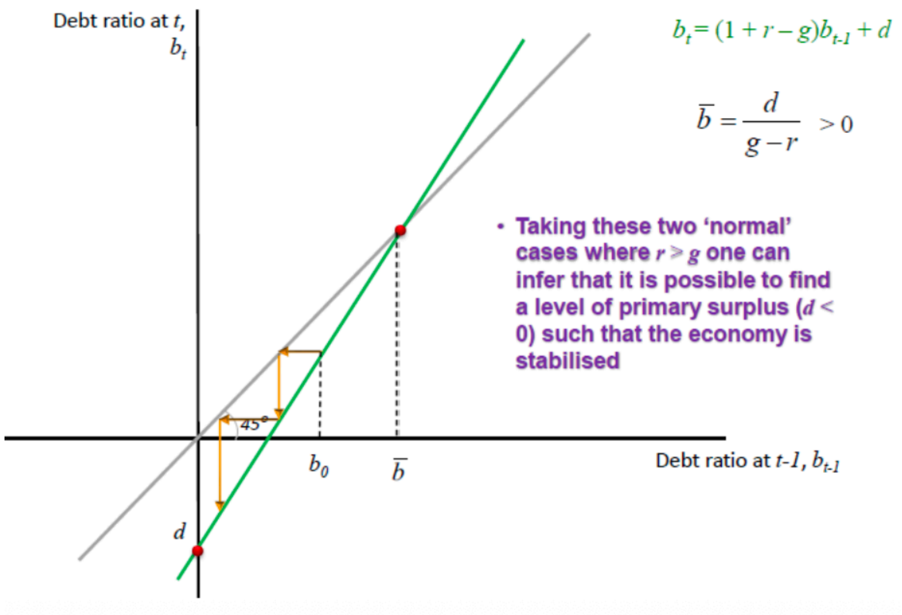
- **Setup:** The government runs a primary deficit and the real interest rate exceeds the growth rate.
- **Debt-to-GDP line:**
  - slope $1 + r - g > 1$ → **steeper than the 45° line**
  - vertical intercept $d > 0$
  - horizontal intercept < 0
  - steady state $\bar{b} = d/(g - r) < 0$ (irrelevant for positive debt).

Dynamics: For any positive initial b_0, the debt ratio **diverges upward at an increasing rate**. There is no convergence.

> ⚠️ This is the dangerous scenario:
> - Debt grows without bound unless fiscal policy changes.
> - To stabilise the debt ratio, the government must switch to a **primary surplus** of sufficient size.

Special case ($d = 0$, $r > g$): debt is still unstable → any positive b_0 generates a rising ratio.

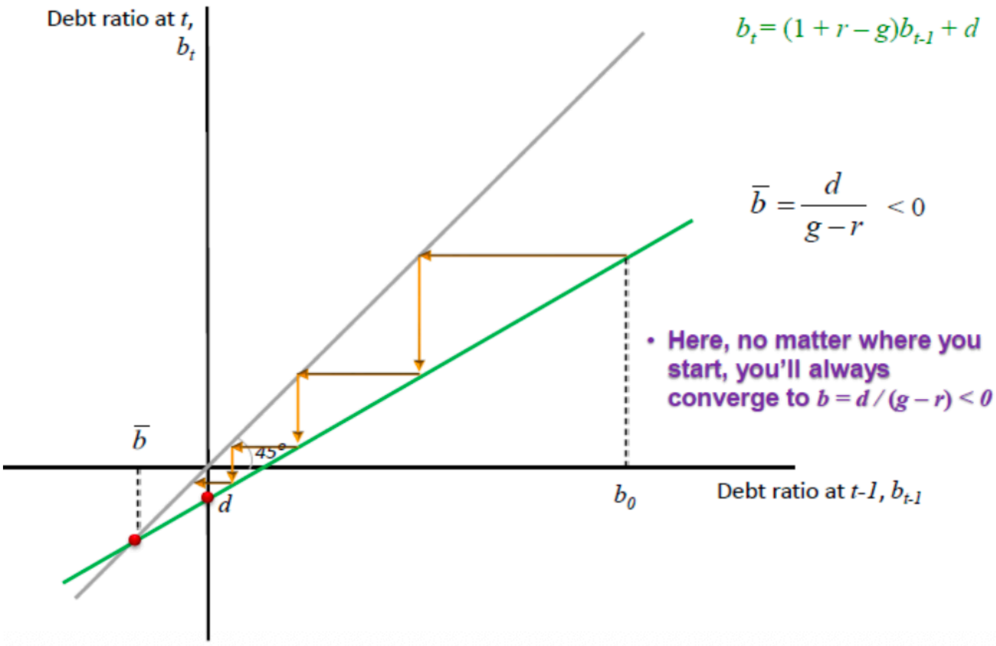
📌 Scenario 3 Primary Surplus ($d < 0$) with $r > g$ → Unstable Equilibrium



- **Setup:** The government runs a primary surplus, but the interest rate still exceeds the growth rate.
- **Debt-to-GDP line:**
  - slope > 1 (steeper)
  - vertical intercept $d < 0$
  - horizontal intercept > 0
  - steady state $\bar{b} = d/(g - r) > 0$.
- **Two competing forces:**
  a. $r > g$ pushes the debt ratio upward
  b. $d < 0$ pushes it downward
  - The outcome depends on b_0 relative to $\bar{b}$:
    - If $b_0 > \bar{b}$: interest effect dominates → **debt ratio rises toward** $+\infty$
    - If $b_0 < \bar{b}$: surplus effect dominates → **debt ratio falls toward** $-\infty$ (government becomes a permanent creditor)

> ⚠️ The equilibrium $\bar{b}$ is **unstable**: any deviation from it leads to divergence in one direction or the other.

📌 Scenario 4 → Primary Surplus ($d < 0$) with $r < g$ → Convergence to Creditor Status



- **Setup:** The government runs a primary surplus and the economy grows faster than the interest rate.
- **Debt-to-GDP line:**
  - slope < 1 (flatter)
  - vertical intercept $d < 0$
  - horizontal intercept > 0
  - steady state $\bar{b} = d/(g-r) < 0$.
- **Dynamics:** Both forces push in the same direction.
  - The debt ratio converges to $\bar{b} < 0$ from any initial b_0
    - the government eventually becomes a **net creditor**.

> ✅ This is the most favourable scenario. The equilibrium is **stable and globally attracting**.

🗂️ Summary of the Four Scenarios

Scenario	d	r vs g	Slope vs 45°	$\bar{b}$	Dynamics	Stability
1	> 0	$r < g$	Flatter	> 0	Converges to $\bar{b}$	✅ Stable
2	> 0	$r > g$	Steeper	< 0 (irrelevant)	Diverges to $+\infty$	⚠️ Unstable
3	< 0	$r > g$	Steeper	> 0	Depends on b_0 vs $\bar{b}$	⚠️ Unstable
4	< 0	$r < g$	Flatter	< 0	Converges to $\bar{b}$	✅ Stable

Special case — Exercise 20 ($r = g$, $d > 0$): The slope equals exactly 1 — the debt-to-GDP line is **parallel to the 45° line**. There is **no steady state**, and the debt ratio grows by exactly d each period: $b_t = b_{t-1} + d$.

4. ⚠️ Dangers of High/Unsustainable Debt

High debt is dangerous for several interconnected reasons.

- **Higher future taxes.**

- To service its debt, the government must eventually raise taxes or cut spending, reducing private-sector welfare.

- **Default risk and the risk premium.**
  - If investors fear that the government may not fully repay its debt, they demand a **higher interest rate** as compensation. If p is the **perceived probability of default**:

$$(1-p)\cdot i_{\text{risky}} + p \cdot 0 = i_{\text{safe}} \quad \implies \quad i_{\text{risky}} = \frac{i_{\text{safe}}}{1-p}$$

During the **Greek crisis (2012)**, 10-year bond yields reached 25% against German yields near 2%, implying markets assigned an implied default probability of roughly 92%.

- **The spread.**
  - The **spread** is the difference in bond yields between a country and the benchmark most creditworthy country (Germany in Europe).
  - In normal times sovereign bond yields remain below 6%.
  - Spreads consistently above 6–8 percentage points signal acute distress.
- **Cost of stabilisation.**
  - A higher r directly increases the primary surplus needed to stabilise the ratio. From the change equation, stability requires:
    - $d = -(r-g)\,b_{t-1} < 0 \quad \text{(primary surplus)}$
      - from: $\Delta b_t = (r-g)b_{t-1} + d$ with $\Delta b_t = 0$

If r rises due to default fears, the required surplus rises too

- demanding cuts in G and/or increases in T that are politically costly and contractionary.

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5. 🔴 The Debt Spiral

The most dangerous outcome of high debt is a **self-reinforcing feedback loop,** a **debt spiral,** that can lead to default even if the initial perceived risk was completely unfounded.

The mechanism:

1. Investors perceive heightened default risk → **demand higher** r
2. Higher r raises the interest burden → larger primary surplus needed to stabilise b
3. Government attempts fiscal consolidation ($G\downarrow$ and/or $T\uparrow$) → contractionary effect → $g\downarrow$
4. Lower g worsens the $(r-g)$ term → debt ratio rises further
5. This validates investors' fears → r rises more

> **Debt spiral:** $\uparrow$ deficit $\Rightarrow \uparrow b \Rightarrow \uparrow r \Rightarrow$ larger required surplus $\Rightarrow$ recession $\Rightarrow \downarrow g \Rightarrow \uparrow b$...

This is a **self-fulfilling crisis**: **even if the original fear was unfounded, the actions it triggers make it come true.**

- **The 2012 Italian/Spanish crisis** illustrates the mechanism.
  - Italian bond spreads over German Bunds rose from ~2% in March to ~5% by July 2012.
  - The spiral was broken only when ECB President **Mario Draghi** announced on 26 July 2012 that the ECB would do **"whatever it takes"** to preserve the euro, signalling readiness to buy sovereign bonds via the **Outright Monetary Transactions (OMT)** programme.
  - Markets believed the commitment, spreads fell, and the ECB never had to actually intervene.

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6. 💸 Debt Default

If the debt spiral cannot be contained, the government may **default** → fail to repay part or all of what is owed.

Types of default:

- **Partial default (haircut)**:
  - Creditors receive only a fraction of their claim.
  - A **30% haircut** means they get 70 cents on the euro.
  - The **Greek restructuring of 2012** involved a ~50% haircut on private creditors.
- **Debt restructuring / rescheduling**:
  - Interest payments are deferred (not cancelled), or maturities are extended.
  - Also called *private sector involvement*, a euphemism for a negotiated loss imposed on creditors.

Why governments are reluctant to default:

- Default carries severe costs:
  - Domestic **pension funds** and **banks** holding government bonds are damaged, causing broader economic disruption.
  - The country's **international reputation is lost**, and access to capital markets may be cut off for years.
  - Future **borrowing costs remain elevated** even after restructuring.

Despite these costs, default can become unavoidable when sustained austerity is politically or economically infeasible.

📝 Exercises Recap

Exercise 20 ($r = g$, $d > 0$): Slope $= 1$; debt-to-GDP line is parallel to the 45° line. No steady state. Debt ratio grows linearly: $b_t = b_{t-1} + d$.

Exercise 21 ($r = 0.03$, $g = 0.05$, Scenario 1):

- (a) Stabilise b at 1: $0 = (0.03 - 0.05)(1) + d \Rightarrow d = 0.02$ (2% primary deficit/GDP is the maximum sustainable level).
- (b) If d was 5%, to bring it to 2% requires $(G_t - T_t)$ to fall → **contractionary fiscal policy** ($G \downarrow$ and/or $T \uparrow$).

Exercise 22 (graph-reading): Slope < 1 → $g > r$. Vertical intercept > 0 → primary deficit. Two ways to stabilise at b^0_{t-1}: (1) shift the line up by raising d ($G \uparrow$ or $T \downarrow$); (2) rotate the line by increasing the slope ($r \uparrow$ via contractionary monetary policy, or $g \downarrow$).

Exercise 23 ($r = -0.04$, $g = 0$, $d = 0.04$): Steady state $\bar{b} = 0.04/(0 - (-0.04)) = 1$. A one-period rate hike to $r' = -0.01$ at $t = 1$ steepens the line, pushing $b_1 > \bar{b}$. When r returns to -0.04 from $t = 2$, the debt ratio converges back to $\bar{b} = 1$ (stable equilibrium).

Exercise 24 ($g = 0.01$, $r = 0.06$, $b_{t-1} = 1.20$, $d = -0.03$):

- (a) $b_t = (1.05)(1.20) - 0.03 = 1.23$
- (b) Stabilise at 1.20: $0 = (0.06 - g)(1.20) - 0.03 \Rightarrow g = 3.5\%$

Exercise 25 ($r = 0.03$, $g = 0$, $b_0 = 1$, $d = 0.03$): $b_1 = (1.03)(1) + 0.03 = 1.06$. To stabilise at $t = 2$: $d' = -(0.03)(1.06) = -0.0318$ (primary surplus of ~3.18% of GDP).

Exercise 26 ($r = -0.02$, $g = 0$, $d = 0.03$): Steady state $\bar{b} = 0.03/0.02 = 1.5$. Tax cut at $t = 1$ only → $d' > d$, line shifts up, $b_1 > \bar{b}$. Taxes restored at √x Invalid equation read off it. Since $g > r$, the equilibrium is stable: b converges back to $\bar{b} = 1.5$.

🗂️ Recap Table

Concept	Key formula / result	Remark
Budget constraint (levels)	$B_t - B_{t-1} = rB_{t-1} + G_t - T_t$	Change in debt = interest payments + primary deficit
Budget constraint (rewritten)	$B_t = (1+r)B_{t-1} + G_t - T_t$	Level of debt next period
Change in debt-to-GDP ratio	$b_t - b_{t-1} = (r-g)b_{t-1} + d$	$d = (G_t - T_t)/Y_t$; key sustainability equation
Debt-to-GDP line	$b_t = (1+r-g)b_{t-1} + d$	Slope $= 1+r-g$; vert. int. $= d$; horiz. int. $= -d/(1+r-g)$
Steady state	$\bar{b} = d/(g-r)$	Where $\Delta b = 0$; requires $g \neq r$
Stable: $g > r$	Slope < 1; converges to $\bar{b}$ regardless of b_0	Scenarios 1 and 4
Unstable: $r > g$	Slope > 1; diverges (or depends on b_0 vs $\bar{b}$)	Scenarios 2 and 3; primary surplus required
The spread	Difference in bond yields vs Germany (benchmark)	Above 6% signals distress; Greek 2012 spread ≈ 23 pp
Debt spiral	$\uparrow b \Rightarrow \uparrow r \Rightarrow$ more consolidation needed $\Rightarrow \downarrow g \Rightarrow \uparrow b$	Self-fulfilling; broken by Draghi "whatever it takes" July 2012
Partial default (haircut)	Creditors receive fraction of face value	Greece 2012: ~50% haircut on private sector debt
Debt restructuring	Interest deferred, maturity extended; debt not cancelled	Also called "private sector involvement"

Great Recession

☑ Reviewed ☐

BAG Chapter 6, Sections 6.3 & 6.5 · Chapter 24, Section 24.4

🏠 1. The Housing Problem

The crisis originated in the **US housing market** and evolved through three successive stages:

Housing Problem → **Financial Crisis** → **Macroeconomic Crisis**

The Housing Bubble

Source: Case-Shiller Home Price Indices, 10-city home price index, http://www.standardandpoors.com/indices/main/en/us

- Throughout the early 2000s, **US housing prices rose sharply**, tracked by the **Case-Shiller Index** (normalised to 100 in January 2000; peaked at **226 in mid-2006**, then collapsed to **162 by end-2008**).

- Two forces drove the boom:
  - **Unusually low interest rates:**
    - → low mortgage rates → **higher demand for housing**.
  - **Subprime mortgages**:
    - loans extended to borrowers with low credit ratings, at higher interest rates to compensate for risk. By 2006, roughly **20% of all US mortgages** were subprime.

Why Did Subprimes Seem Safe at the Time?

- Lenders assumed housing prices would keep rising, even if a borrower defaulted, the house could be sold at a profit.
- Many loans featured low **"teaser rates"** (initial low interest payments that increased sharply later), which borrowers did not fully understand.
- Housing prices had not fallen even during the 2000–01 recession — this seemed like historical evidence of safety.

The Collapse

- From **2006 onwards**, prices fell. Many mortgages went **underwater**: the outstanding mortgage value exceeded the value of the house.
- Borrowers with teaser-rate loans found payments unaffordable even without the price fall.
- Mass **defaults** followed.
  - By **mid-2008**, losses on mortgages were estimated at **$300 billion ≈ 2% of US GDP**.

This initial **shock alone was manageable**, the problem was the **amplification** through the financial system.

🏦 2. The Role of Financial Intermediaries (§6.3)

Financial intermediaries (banks, hedge funds, mortgage companies) **receive funds from investors and lend them to others**, charging a higher interest rate than they pay in order to earn a profit. Understanding their balance sheet structure is essential to understanding the crisis.

Bank Balance Sheet: Key Concepts

A simplified bank balance sheet:

Assets	Liabilities
100	Liabilities: 80
	Capital: 20

Two critical ratios follow from this:

$$\text{Capital Ratio} = \frac{\text{Capital}}{\text{Assets}}$$

$$\text{Leverage Ratio} = \frac{\text{Assets}}{\text{Capital}} = \frac{1}{\text{Capital Ratio}}$$

The leverage ratio is the inverse of the capital ratio. A bank with assets 100 and capital 20 has a capital ratio of 20% and a leverage ratio of 5.

The Leverage Trade-off

A **higher leverage ratio** has two opposing effects:

① Higher expected profit per unit of capital

With assets = 100, return on assets = 5%, return on liabilities = 4% (→ what you have to pay to lenders):

- If capital = 20 (leverage = 5):
  - $\text{Profit} = 100 \times 5\% - 80 \times 4\% = 1.8 \Rightarrow \text{Profit/capital} = \frac{1.8}{20} = 9\%$
- If capital = 10 (leverage = 10):
  - $\text{Profit} = 100 \times 5\% - 90 \times 4\% = 1.4 \Rightarrow \text{Profit/capital} = \frac{1.4}{10} = 14\%$

② Higher risk of insolvency

- A bank is **solvent** if assets > liabilities; **insolvent** otherwise.
- With capital = 20 and liabilities = 80: assets must fall below **80** before insolvency.
- With capital = 10 and liabilities = 90: assets need only fall below **90**
  - insolvency risk is much higher.

Banks must choose a leverage ratio that **balances profit against insolvency risk**. In the 2000s, they systematically chose too high.

Leverage and Lending: The Deleveraging Spiral

Suppose asset values fall (e.g., from 100 to 90 (bad loans)). Capital falls from 20 to **10**, and leverage rises from 5 to **9**.

The bank, still solvent but more fragile, will try to **restore its capital ratio**. Its options:

- **Raise new capital**
  - difficult in a crisis, when investors are unwilling to commit funds.
- **Call back loans**
  - reduce assets to restore the ratio
  - e.g., call back loans worth 40 → assets fall to 50, liabilities to 40, capital ratio = 10/50 = 20% ✓. But this means a **sharp reduction in lending**.
- **Sell assets at fire sale prices** (see below)

If assets fall all the way from 100 to **70**, the bank becomes **insolvent** and goes bankrupt. Borrowers lose access to credit entirely.

Liquidity: The Fire-Sale Mechanism

Even if a bank's assets have not actually declined, **doubts** about their value can trigger a crisis:

1. Investors become nervous → they **withdraw their funds** (demand deposits can be pulled on short notice).
2. The bank must come up with cash immediately, but **loans cannot easily be called back** (borrowers have already spent the money).
3. The bank tries to sell loans to other banks, but other banks find it hard to assess their value → loans can only be sold at **fire sale prices** (far below true value).
4. Fire sales reduce asset values (also of the assets that are not sold) → capital falls → leverage rises → more fire sales required → the bank may become insolvent **even if it was fully solvent to begin with**.

The key asymmetry: **assets are illiquid** (hard to sell quickly at fair value); **liabilities are liquid** (demand deposits can be withdrawn immediately). This mismatch is the source of fragility.

- the more illiquid the assets and the more liquid the liabilities = the higher the risk of this mechanism

Bank Runs

This liquidity mismatch gives rise to **bank runs**: if enough depositors fear a bank is failing and withdraw their funds simultaneously, the bank is forced into fire sales and may collapse, **even if all its underlying loans were sound**. The **fear becomes self-fulfilling**.

Policy responses to bank runs:

- **Deposit insurance**: government guarantees deposits up to a ceiling ($250,000 in the US since 2008; €100,000 per account in the EU). Removes depositors' incentive to run.
    - If the bank fails, you still get your deposited money up to a limit
- **Central bank liquidity provision** (lender of last resort): the central bank lends to banks against the value of their assets, preventing forced fire sales.
    - If a bank is basically solvent but temporarily short of cash, the central bank lends it cash so it does not have to panic-sell assets.
        - CB lends money, and uses bank's assets as collateral
    - DIFFERENCE BETWEEN "ILLIQUID" AND "INSOLVENT"
        - illiquid
            - "I can pay you, but not immediately."
        - insolvent
            - "Even if I sell everything, I cannot pay everyone."

Both instruments have limits:

- deposit insurance can encourage excessive risk-taking (moral hazard)
- liquidity provision requires the central bank to distinguish illiquid-but-solvent from genuinely insolvent banks — difficult in a crisis.

💥 3. From a Housing Problem to a Financial Crisis (§6.5)

The $300 billion in mortgage losses was only 2% of US GDP. What turned it into the deepest recession since WWII was **amplification** through three structural features of the financial system.

Factor 1: High Leverage

Banks were already operating with high leverage ratios before the crisis, for three reasons:

- **Underestimation of risk**: in good times, banks (like individuals) tend to discount tail risks.
- **Incentive problems**: bonus systems rewarded managers for high returns without penalising bankruptcy risk.
- **Regulatory arbitrage via SIVs** (see below).

Factor 2: Securitisation and Illiquid Assets

An important financial innovation of the 1990s–2000s was **securitisation**: instead of holding **loans** on their own balance sheets, banks **bundled them into tradeable securities**.

Mortgage-Backed Security (MBS):

- A title to the returns from a **bundle of (often tens of thousands of) mortgages**.
- Investors who would not hold individual mortgages were willing to buy MBSs, expanding the supply of mortgage funding.

Collateralised Debt Obligations (CDOs):

- (bundles of debt products, can contain MBS, corporate loans or bonds...)
- Securities built from bundles of MBSs or other assets, divided into tranches:
    - **Senior securities**: paid first from the bundle's returns; lower risk.
    - **Junior securities**: paid only after senior claims are met; higher risk, higher yield.

- **CDO²**: CDOs composed of other CDOs → extreme complexity.

Why securitisation created fragility:

1. Once a bank sold a mortgage into a securitisation bundle, it no longer bore the credit risk
   - selling the CDOs investors bear the risk
   - → **reduced incentive to screen borrowers carefully**.
2. **Rating agencies** failed to properly assess the risk embedded in MBSs and CDOs.
   - When underlying mortgages went bad, the c**omplexity of these instruments made valuation nearly impossible.**
3. Investors assumed the worst and stopped holding these **toxic assets**
   - → assets became **extremely illiquid** (could only be sold at fire sale prices, if at all).
     - illiquid because at one point nobody wanted to buy them anymore

Factor 3: Wholesale Funding and Liquid Liabilities

Banks increasingly funded themselves not through retail deposits but through **wholesale funding**: **short-term borrowing from other banks and institutional investors**.

- Wholesale creditors are **not protected by deposit insurance**.
- When they grew concerned about bank solvency, they simply **refused to roll over their loans** → banks faced an **immediate liquidity crisis**.
- This is the **modern equivalent of a bank run**, except it affects wholesale funders rather than retail depositors.

Structured Investment Vehicles (SIVs) exploited all three factors simultaneously:

- entities created by banks to hold MBSs and CDOs **off-balance-sheet** (bypassing capital requirements → higher effective leverage).
- Funded entirely through **wholesale short-term debt** → fully liquid liabilities.
- Held **complex, illiquid assets**.

- **Banks provided an implicit guarantee**: if the SIV ran into trouble, the bank would cover it. When SIVs collapsed, banks were exposed.

The Perfect Storm

HIGH LEVERAGE + ILLIQUID ASSETS + LIQUID LIABILITIES = Financial Crisis

The mechanism:

1. **Housing prices fall** → subprime mortgages default → MBSs and CDOs embedded in SIV **portfolios lose value**.
2. Wholesale funders, unable to assess asset values, **refuse to roll over debt**.
3. SIVs and banks are forced to sell assets → **fire sales → asset prices fall further**.
4. Fire sales reduce the value of similar assets held by other intermediaries → their capital ratios deteriorate → more forced sales → prices fall further (**self-reinforcing spiral**).
5. On **15 September 2008**, **Lehman Brothers** (assets > $600 billion) declared bankruptcy. Financial participants concluded that most other institutions were at risk.
6. By late September 2008, **the financial system was paralysed**: banks stopped lending to each other and to the real economy.

From Financial Crisis to Macroeconomic Crisis

The financial crisis transmitted to the real economy through these main channels:

① Higher borrowing rates

- Interest rates on AAA-rated bonds rose to ~7%; on BBB-rated bonds to ~10%.
- Small firms entirely dependent on bank credit found borrowing nearly impossible.
  - loans were very expensive
- In IS-LM terms: the risk premium x rose sharply → borrowing rate $r + x$ increased → investment fell → **IS shifted left**.

② Collapse in confidence

- Consumer confidence (indexed to January 2007 = 100) fell to a low of **22 in early 2009** — far below any historical precedent.
- Lower confidence → lower consumption ($C \downarrow$) and investment ($I \downarrow$) → **further leftward shift of IS.**

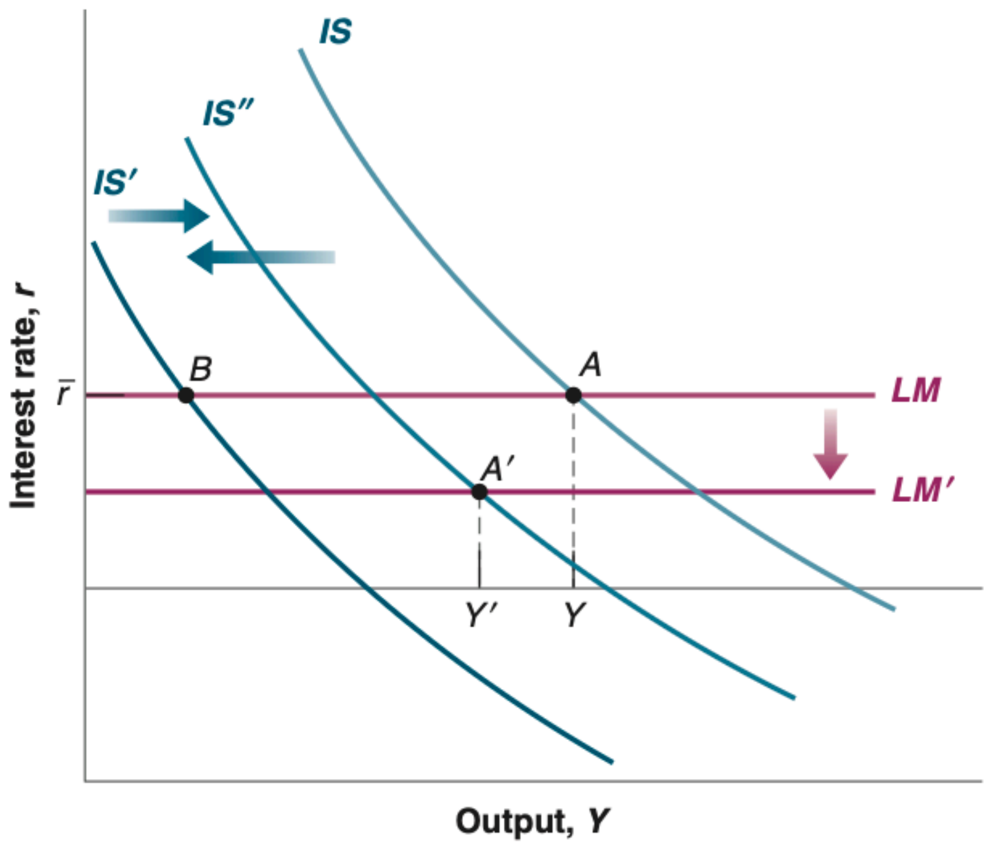
③ International spillovers

- European banks had directly purchased US MBSs and CDOs → exposed to US housing losses.
- **Rising US interest rates transmitted to European rates.**
- Falling US demand → collapse in US imports → fall in exports from trading partners.
- House prices had also risen sharply in the UK, Spain, and Ireland → similar domestic amplification.

🏛️ 4. Policy Responses

IS-LM Representation

The crisis is cleanly represented in the extended IS-LM model:



- **Crisis**: IS shifts sharply left (from IS to IS') → without policy, equilibrium moves from A to B (large fall in output).
- **Financial & fiscal policy**: partially shifts IS back right (from IS' to IS'').
- **Monetary policy**: shifts LM down (from LM to LM') → equilibrium moves to A'.
- Even combined, policies were insufficient to avoid a major recession. **US GDP fell 3.5% in 2009**.

US Policy Response

Financial policies (aimed at stabilising the financial system):

- Federal deposit insurance raised from $100,000 to **$250,000** per account.
- The Fed provided **liquidity facilities**:
  - banks and non-bank **intermediaries could borrow from the Fed using their assets as collateral** → prevented forced fire sales.
- **TARP** (Troubled Asset Relief Program, October 2008, $700bn):

- initially intended to remove toxic assets from balance sheets; later redirected to **recapitalise banks** (government purchased equity stakes), **increasing capital ratios and reducing leverage**. By end of September 2009, $360bn spent, of which $200bn through share purchases.

Monetary policy:

- The Fed began cutting the federal funds rate in summer 2007; by **December 2008** the rate was at **zero** (zero lower bound).
- **Conventional monetary policy was exhausted** → pivot to **unconventional monetary policy** (see §24.4 below).

Fiscal policy:

- **American Recovery and Reinvestment Act** (February 2009): $780 billion in tax cuts and spending increases over 2009–2010.
- US budget deficit rose from **1.7% of GDP in 2007** to **9% in 2010**.

European Policy Response

- **Financial**:
  - Europe was much slower to clean up bank balance sheets. The UK recapitalised banks as early as October 2008; other countries, notably Italy, delayed for years. Italian banks still held ~€350bn in non-performing loans as late as 2015.
- **Fiscal**:
  - Smaller stimulus overall than the US. Countries with high pre-crisis debt (e.g., Italy at ~90% debt/GDP) had little fiscal space; countries with low debt (e.g., Denmark) implemented larger expansions.
- **Monetary**:
  - The Bank of England moved quickly to unconventional policies once rates hit zero. The **ECB** did not begin asset purchases until **March 2015,** over two years after its deposit rate hit zero, significantly delaying the eurozone recovery. The ECB, Swedish Riksbank, and Swiss National Bank eventually lowered rates **below zero**, charging banks a deposit fee to discourage holding reserves.

🔧 5. Unconventional Monetary Policy — Quantitative Easing (§24.4)

The Zero Lower Bound Problem

When the policy rate $r = 0$, conventional monetary policy is exhausted. But **other interest rates remained well above zero**, reflecting **risk premiums** x on various assets. The extended IS-LM borrowing rate is:

$$r_{\text{borrow}} = r + x$$

Conventional monetary policy targets r (the policy rate).

Unconventional monetary policy targets x (the risk premium).

The Logic of *Quantitative Easing (QE)*

The risk premium on an asset is determined by **supply and demand for that asset**:

- If demand for an asset falls (investors become more risk-averse) → $x \uparrow$ → borrowing rates rise.
- If demand increases → $x \downarrow$ → borrowing rates fall.

This holds regardless of whether the buyer is a private investor or the central bank.

By **purchasing assets directly** (other than short-term bonds), the central bank increases demand for those assets, compresses their risk premiums, and reduces the corresponding borrowing rates, stimulating investment and spending even when the policy rate cannot be cut further. The purchases are financed by **money creation** (issuing reserves).

QE Programmes in the United States

Programme	Date	Assets Purchased	Objective
QE1	November 2008	Mortgage-backed securities (MBS)	Restore functioning in the dysfunctional MBS market; compress mortgage premium
QE2	November 2010	Long-term Treasury bonds	Reduce term premium on long-term government bonds

Programme	Date	Assets Purchased	Objective
QE3	September 2012	Further MBS purchases	Reduce mortgage costs; support housing market recovery

The **monetary base** expanded from ~$850bn (6.6% of GDP) in September 2008 to **~$4,000bn (22% of GDP) by 2015** — more than quadrupled. Most of the increase sat as **bank reserves** at the Fed.

Assessment of QE

- **QE1**: broadly judged as highly effective. The MBS market was genuinely dysfunctional; direct intervention dramatically reduced premiums.
- **QE2 & QE3**: more controversial. These intervened in markets that were no longer dysfunctional. They reduced term premiums on long-term bonds, but by how much is debated.

> **General conclusion**: QE had *some* effect on borrowing rates and thereby on output, so the zero lower bound does not make monetary policy fully impotent. However, QE operates through more complicated and less reliable channels than conventional policy.

The Corridor System (Post-ZLB Exit)

When the Fed raised rates again from end-2015, it adopted a **corridor system**. The Fed sets two rates:

- **Interest rate on reserves (floor)**:
  - The **rate at which banks effectively lend to the Fed**.
  - No bank will lend to another bank at a rate below this.
- **Discount rate (ceiling)**:
  - The **rate at which banks borrow from the Fed.**
  - No bank will borrow from another bank above this.

The federal funds rate is determined in the interbank market but constrained to remain within this corridor. The **floor rate on reserves has become the primary policy tool**.

🗂️ Recap: The Chain of the Crisis

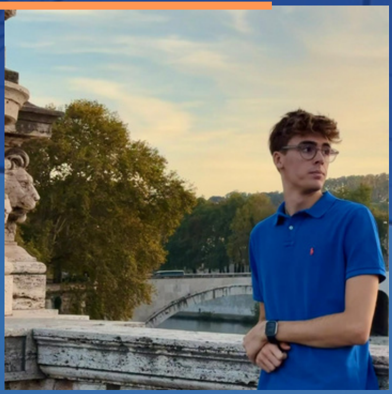
Step	What Happened	Key Mechanism
1. Housing bubble	Prices rise 2000–06; subprime lending grows	Low rates + lax standards + false price expectations
2. Price collapse	Prices fall from 2006; mortgages go underwater	Defaults surge; lender losses ~$300bn
3. Asset illiquidity	MBSs/CDOs become impossible to value	Toxic assets; rating agency failure; securitisation opacity
4. Leverage spiral	Banks forced to deleverage → cut lending or sell assets	Fire sales → asset prices fall → more forced selling
5. Liquidity crisis	Wholesale funders withdraw; SIVs collapse	Liquid liabilities, illiquid assets; modern bank runs
6. System paralysis	Lehman bankruptcy (Sept 2008); interbank market freezes	Contagion; loss of confidence across all institutions
7. Macro crisis	$r + x uparrow$; confidence collapses; IS shifts left	Credit squeeze hits I; wealth effect hits C
8. Policy response	TARP, liquidity facilities, fiscal stimulus, rate cuts	IS partially shifted back; LM shifted down
9. ZLB hit	Federal funds rate = 0 by December 2008	Conventional policy exhausted
10. QE	Fed buys MBSs and Treasury bonds	$x \downarrow \Rightarrow r + x \downarrow \Rightarrow I \uparrow$ → partial recovery

📌 Key Terms

Term	Definition
Capital ratio	Capital / Assets
Leverage ratio	Assets / Capital
Subprime mortgage	Loan to a high-risk borrower at a higher interest rate
Underwater mortgage	Mortgage value exceeds house value
Fire sale	Forced asset sale at prices far below true value
SIV (Structured Investment Vehicle)	Off-balance-sheet entity created by banks to hold securities, funded via wholesale debt

Term	Definition
MBS (Mortgage-Backed Security)	Security based on a bundle of mortgages
CDO (Collateralised Debt Obligation)	Security structured from tranches of MBS or other debt
Toxic assets	Assets so complex their value cannot be assessed
Wholesale funding	Bank financing via short-term borrowing from other banks/investors (not retail deposits)
TARP	US government programme to recapitalise banks ($700bn, October 2008)
Quantitative Easing (QE)	Central bank purchase of non-standard assets to reduce risk premiums when the policy rate is at zero
Zero lower bound	The constraint that the nominal interest rate cannot fall below zero
Corridor system	Post-QE framework where the Fed sets a floor (interest on reserves) and ceiling (discount rate) for the federal funds rate

FOR DOUBTS OR SUGGESTIONS ON THE HANDOUTS



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