UNILAB NETWORK

HANDOUT

FINANCIAL ECONOMICS

2022-2023 EDITION

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FINANCIAL ECONOMICS FORMULAE AND KEY FACTS

Chapter 5 (5.2,5.4-5.7)

Effective Annual Rate

$$\mathrm{EAR} = \left(1 + \frac{\mathrm{APR}}{\mathrm{n}}\right)^{\mathrm{n}} - 1 = \left(1 + \mathrm{r_f(n)}\right)^{\frac{1}{\mathrm{n}}} \text{ where } \mathrm{r_f(n)} = \frac{\mathrm{FV}}{\mathrm{PV}} - 1$$

Annual Percentage Rate

$$\mathrm{APR} = \frac{(1 + \mathrm{EAR})^{\mathrm{n}} - 1}{\mathrm{n}} = \mathrm{n} \times \mathrm{r_f(n)}$$

Holding Period Return

$$\mathrm{HPR} = \frac{\text{Ending price of share} - \text{Beginning price} + \text{Dividends}}{\text{Beginning Price}}$$

Expected Return and Standard Deviation

$$E(r) = \sum_{s} p(s) r(s) \qquad \& \qquad \sigma^2 = \sum_{s} p(s)\big(r(s) - E(r)\big)^2$$

Arithmetic and geometric (time-weighted) average returns

$$\text{Arithmetic} = \frac{1}{\mathrm{n}} \sum_{\mathrm{s}=1}^{\mathrm{n}} \mathrm{r(s)} \qquad \text{Geometric; Terminal value} = (1 + \mathrm{g})^{\mathrm{n}} \therefore \mathrm{g} = \mathrm{TV}^{\frac{1}{\mathrm{n}}}$$

The greater the volatility in rates of return, the greater the discrepancy between arithmetic and geometric averages. If Normally distributed; $\mathrm{E(Geometric)} = \mathrm{E(Arithmetic)} - \frac{1}{2}\sigma^2$
Sharpe Ratio – Reward-to-Volatility ratio (the higher the better i.e. more reward and less risk)

$$\mathrm{S_R} = \frac{\mathrm{r_x} - \mathrm{r_f}}{\sigma_{\mathrm{x}}}$$

Skew, Kurtosis, Value at Risk

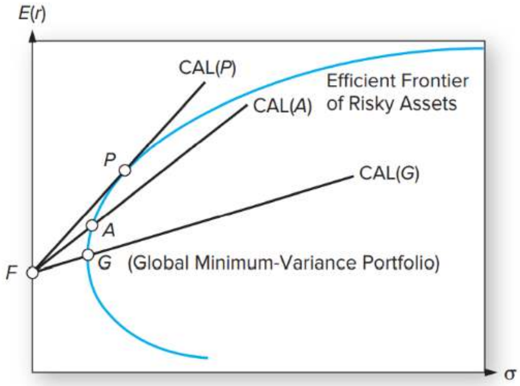
$$\text{Skew} = \text{Average}\left(\frac{(\mathrm{R} - \bar{\mathrm{R}})^3}{\hat{\sigma}^3}\right), \text{Kurt} = \text{Average}\left(\frac{(\mathrm{R} - \bar{\mathrm{R}})^4}{\hat{\sigma}^4}\right) - 3$$

$$\mathrm{VaR}(1\%, \text{normal}) = \text{Mean} - 2.33\mathrm{SD}$$

Value at risk is the loss corresponding to a very low percentile of an entire return distribution

Chapter 6 (6.1-6.4)

Utility of risk aversion



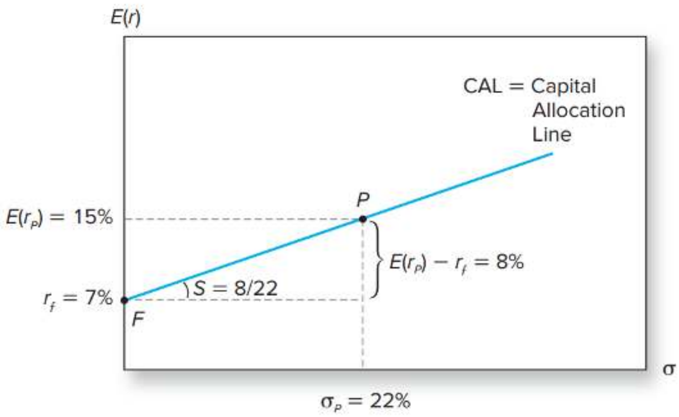
$$U = E(r) - \frac{1}{2}A\sigma^2$$

Risk Neutral: A = 0, Risk Averse: A > 0, Risk Loving: A < 0

Construction of a portfolio with one risky and one risk free asset

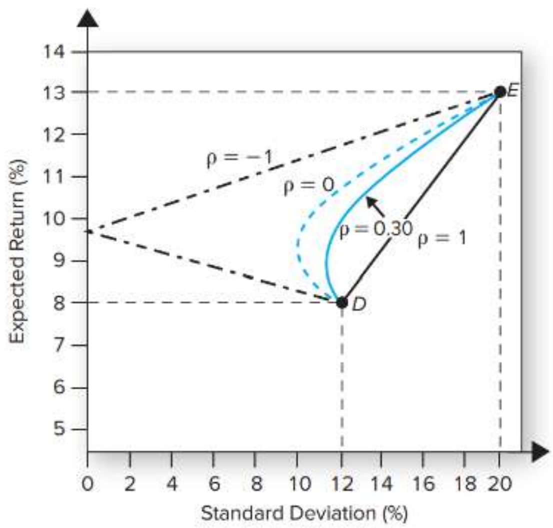
$$r_p = w_1 r_a + (1 - w_1) r_f$$

Capital Allocation Line (CAL)



Shows all the risk-return combinations available to investors. The slope is the sharpe ratio.

Chapter 7 (7.1-7.4)



$$r_p = w_e r_e + w_d r_d \quad \& \quad E(r_p) = w_d E(r_d) + w_e E(r_e)$$

$$\sigma_p^2 = w_d^2\sigma_d^2 + w_e^2\sigma_e^2 + 2 w_d w_e \rho_{de}\sigma_d\sigma_e \quad Cov(r_d, r_e) = \rho_{de}\sigma_d\sigma_e$$

$$\text{when } \rho_{de} = 1, \sigma_p^2 = (w_d\sigma_d + w_e\sigma_e)^2$$

Using the formula for risk aversion utility we find the weight of debt:

$$w_d = \frac{E(r_d) - E(r_e) + A(\sigma_e^2 - \sigma_d\sigma_e\rho_{de})}{A(\sigma_d^2 + \sigma_e^2 - 2\sigma_d\sigma_e\rho_{de})}$$

We can also use it to find the weight of Equity (i.e. invested in the portfolio and not the rf)

$$w_e = \frac{E(r_p) - r_f}{A\sigma_p^2}$$

Chapter 8 (8.1-8.3, 8.5)

Input list of the markowitz model

- N=50= Number of expected returns
- N=50= Number of variances (1,1)(2,2)(3,3)
- $\frac{n^2-n}{2}$= Number of covariances

Systematic vs Firm-Specific Risk

$\text{Total Risk} = \sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma^2(e_i)$

Where the beta is the sensitivity to market risk and ei is the firm specific risk

$\text{Cov}(r_i, r_j) = \beta_i\beta_j\sigma_m^2$ if firm specific risk is uncorrelated

Index Model regression equation

Alpha is the security's expected excess return when the market excess return is zero

$$R_i(t) = \alpha_i + \beta_i R_m(t) + e_i(t)$$

$$E(R_i) = \alpha_i + \beta_i E(R_m)$$

$$\text{Corr}(r_i, r_j) = \text{Corr}(r_i, r_m) \times \text{Corr}(r_j, r_m) = \frac{\beta_i\beta_j\sigma_m^2}{\sigma_i\sigma_j}$$

Estimate of Beta

$$\frac{Estimated\ value - Hypothesized\ Value}{Standard\ Error}$$

Optimal Risky Portfolio in the Single-Index Model

$$\alpha_p = \sum_{i=1}^{n+1} w_i\alpha_i, \qquad \beta_p = \sum_{i=1}^{n+1} w_i\beta_i, \qquad \sigma^2(e_p) = \sum_{i=1}^{n+1} w_i^2\sigma^2(e_i)$$

$$E(r_p) = \alpha_p + E(r_m)\beta_p \qquad \therefore \qquad S_p = \left(\frac{E(r_p)}{\sigma_p}\right)$$

Chapter 9

Market Price of Risk

$$\frac{Market\ Risk\ Premium}{Market\ Variance} = \frac{E(r_m)}{\sigma_m^2}$$

Mean Beta Relationship

Overpriced stocks lie below the SML (+ve alpha) and vice versa

$$E(r_p) = r_f + \beta_p(E(r_m) - r_f)$$

Chapter 14

Bond Value

$$Bond\ Value = \sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \frac{Par\ Value}{(1+r)^T}$$

Chapter 15

Forward Rates

$$(1+y_n)^n = (1+y_{n-1})^{n-1} \times (1+r_n)$$
$$(1+f_n) = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}}$$

Chapter 16 (16.1-16.3)

Duration

$$\frac{\Delta P}{P} = -D \times \frac{\Delta(1+y)}{1+y} \cong -D \times \Delta y$$
$$Duration = \frac{\sum_{t=1}^{T} \frac{t * C_t}{(1+r)^t}}{\sum_{t=1}^{T} \frac{C_t}{(1+r)^t}}$$

Chapter 18 (18.1-18.3)

Dividend Discount Model

$$V_0 = \frac{D_1 + P_1}{1+i}\ and\ so\ V_1 = \frac{D_2 + P_2}{1+i}\ therefore\ V_0 = \frac{D_1}{1+i} + \frac{D_2 + P_2}{(1+i)^2}$$
$$V_0 = \frac{D_1}{k-g} \qquad g = ROE \times b \qquad D_1 = E_1(1-b) \qquad P_t = P_0(1+g)^t$$

Where b is the ploughback ratio

Chapter 20

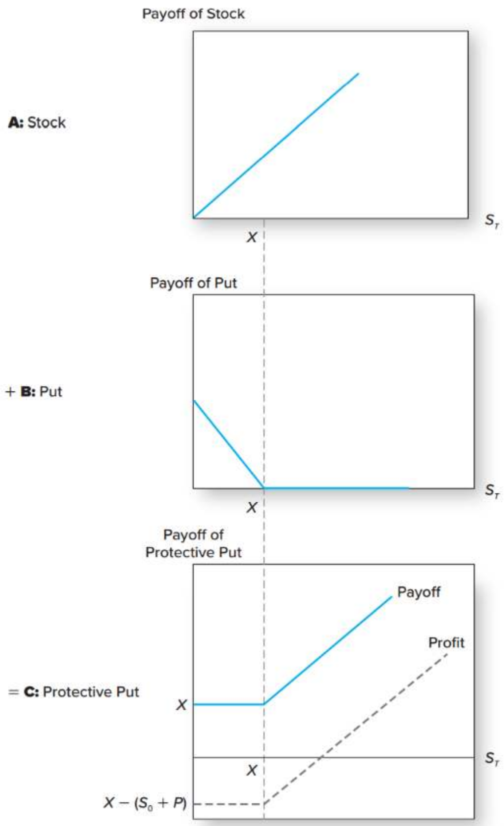
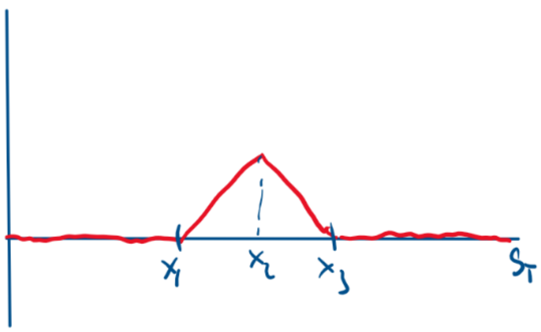


Figure 20.6 Value of a protective put position at option expiration

Payoff Butterfly



$Call(x_1) - 2call(x_2) + call(x_3)$

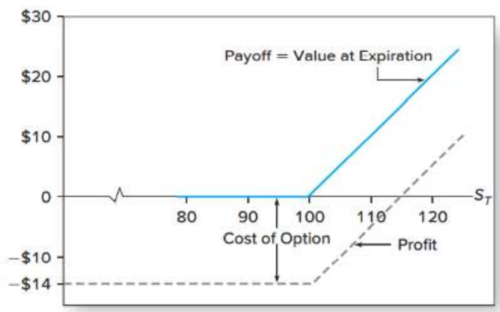


Figure 20.2 Payoff and profit to call option at expiration

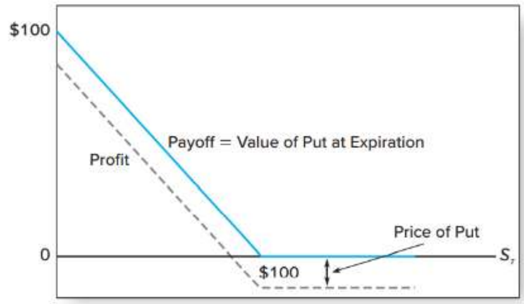


Figure 20.4 Payoff and profit to put option at expiration

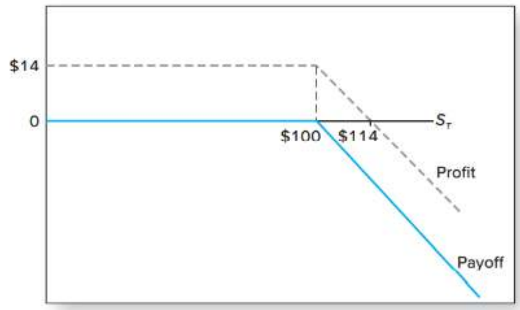


Figure 20.3 Payoff and profit to call writer at expiration

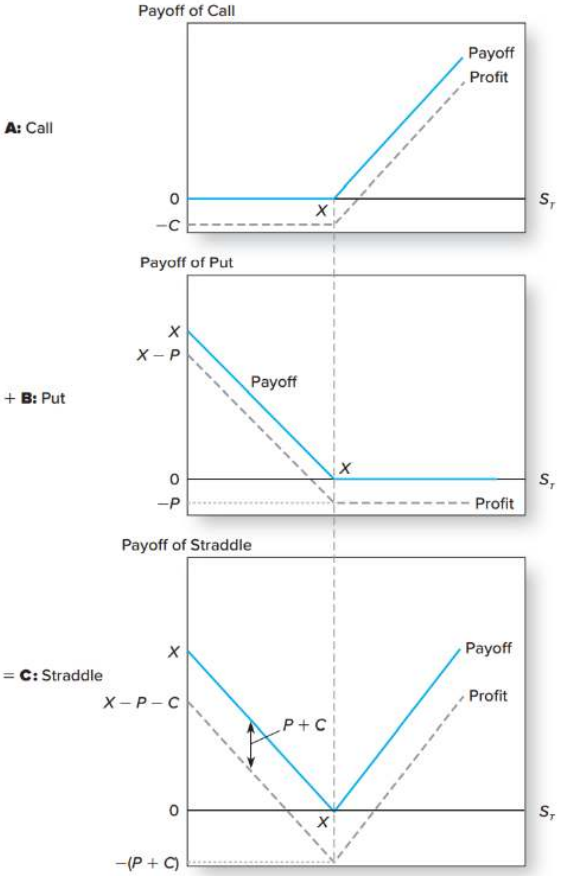


Figure 20.9 Value of a straddle at expiration

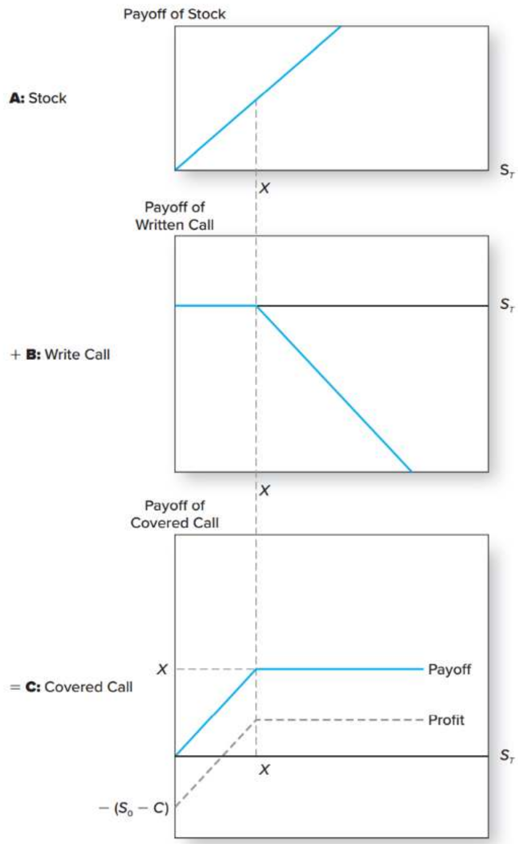


Figure 20.8 Value of a covered call position at expiration

Chapter 21

Hedge Ratio

$$H = \frac{C_u - C_d}{uS_0 - dS_0}$$

Probability

$$q_u = \frac{1 + r_f - d}{u - d}$$

Price of the option giving option payoff

$$\frac{(U_u \times P) + ((1 - P) \times U_d)}{(1 + r_f)}$$

Minimum price of option *derived from Black-Scholes*

$$Max(0, X - Xe^{-r_f T})\ where\ X\ is\ the\ stock\ price$$

Black-Scholes-Merton Formula

$$C_0 = S_0 e^{-\delta T} N(d_1) - Xe^{-rT} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Call option is said to be in teh money of N(d1) is approximately = N(d2) and 1. The put option is therefore almost certainly out of the money

$$\Delta = \frac{\vartheta C_0}{\vartheta S_0} = e^{-\delta T} N(d_1)$$

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